1. 請以羅斯表(Routh-criterion) 判斷下列轉移函數是否穩定？

$$G(s)=\frac{1}{s^3+2s^2+s+2}$$

Ans:

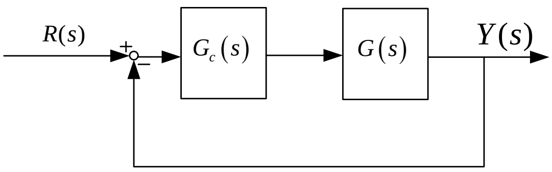
羅斯表

s^3	1	1	
s^2	2	2	$A(s)=2s^2+2$
s	0	0	$dA(s)/ds=4s$
s	4		
s^0	2		

檢查羅斯表第一行元素沒有正負變號，但 A(s)=0 解得 $s=\pm j$ ，表示特性方程式有兩個純虛根，所以系統是不穩定的。

2. **有一回授控制系統的方塊圖如下表示，其中受控系統的轉移函數 $G(s)=\frac{1}{s(s^2+s+1)(s+3)}$，控制器轉移函數 $G_c(s)=K$，試求控制器 K 值的範圍，能使閉迴路穩定。**

Ans:



閉迴路轉移函數為

$$\frac{Y(s)}{R(s)}=\frac{K}{s^4+3s^3+3s^2+3s+K}$$

利用特性方程式 $s^4+3s^3+3s^2+3s+K=0$ 所得的羅斯表如下:

s^4	1	3	K
s^3	3	3	
s^2	2	K	
s	$\frac{3K-6}{2}$		
s^0	K		

若希望閉迴路系統穩定，亦即閉迴路極點均在 s 左半平面，則羅斯表第一行元素

必須全部為正(沒有正負變號)，所以 $\frac{3K-6}{2}>0,\ \ \mathrm{K}>0 \Rightarrow\ \ 0<\mathrm{K}<2$

3.考慮單一輸入單一輸出系統 $y'''(t)+y''(t)+6y'(t)+(k-3)y(t)=u(t)$**，求使得系統穩定的 k 值範圍**

Ans:

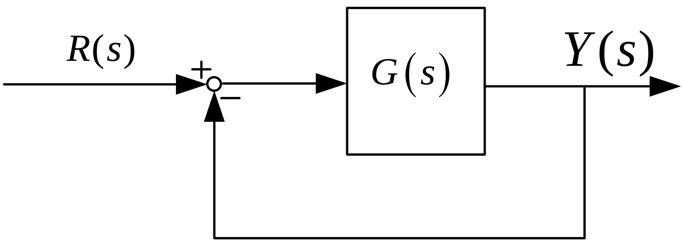
對系統微分方程式取拉式轉換，並令初值為零，則可得系統的轉移函數為

$\frac{Y(s)}{U(s)}=\frac{1}{s^3+s^2+6s+k-3}$　其羅斯表為

s^3	1	6
s^2	1	$k-3$
s	$9-k$	
s^0	$k-3$	

所以 k 值穩定範圍必須滿足 $9-k>0,\ \ k-3>0\ \ \Rightarrow\ \ 3<k<9$

4.考慮閉迴路系統 $G(s)=\frac{1}{s(1+0.5s)(1+s)}$ **，請求出相位邊限(Gain margin)。**

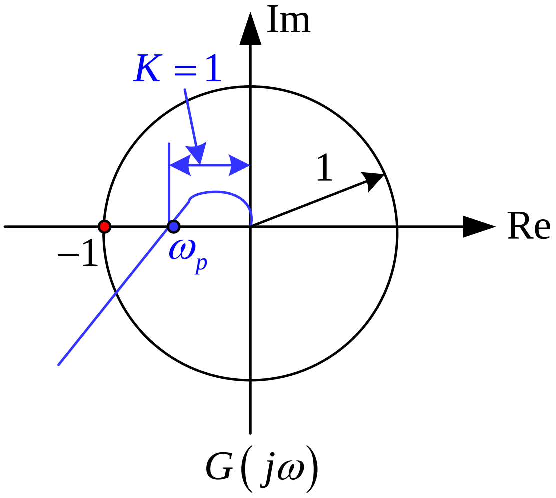


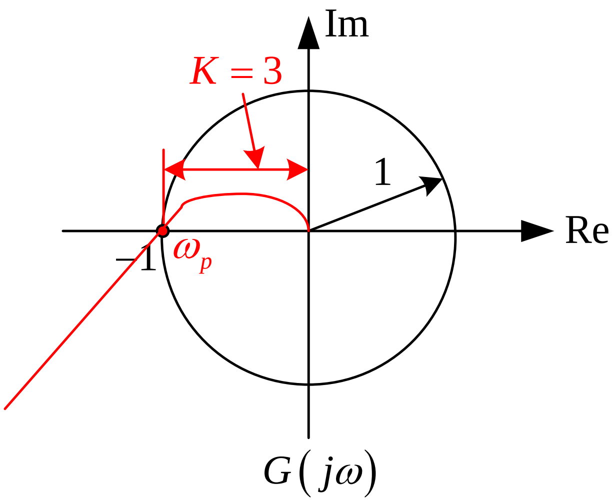
Ans:

令 $G(s)=\frac{K}{s(1+0.5s)(1+s)}$ ，則閉迴路特性方程式為: $s^3+3s^2+2s+2K=0$ ，利用羅斯表判斷閉迴路穩定的 K 值範圍:

s^3	1	2
s^2	3	2K
s	$\frac{6-2K}{3}$	
s^0	2K	

因此閉迴路穩定的 K 值範圍是 0<K<3。而目前系統的 K 值為 K=1，
若由極座標圖來看:

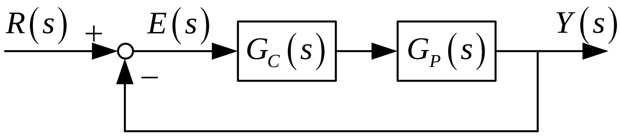




根據 Gain Margin 的物理定義:系統到達臨界穩定前還能增加或減少的增益(gain)倍數 (以 dB 為單位)。
故可求得 $G.M.=20\log 3=9.54\ dB$

5.

Consider the following control problem



where $G_P(s)=\dfrac{s^2+2s+1}{s^2-1}$ and $G_C(s)=\dfrac{K}{s}$.

(a) Determine the range of K that will result in a stable closed-loop system.

(b) What is the close-loop oscillation frequency (write down the unit) for marginally case?

Solution:

(a)

The characteristic equation will be,

$$\Delta(s)=1+G_C(s)G_p(s)=s^3+Ks^2+(2K-1)s+K$$

Using the Routh-Hurwitz table,

$$\begin{array}{lll} s^3 & 1 & 2K-1 \\ s^2 & K & K \\ s^1 & \dfrac{2K^2-2K}{K} & \\ s^0 & K & \end{array}$$

To stabilize the system, K must satisfy the following condition,

$$\begin{cases} K>0 \\ 2K-1>0 \\ 2K^2-2K>0 \end{cases}$$

From the above, we can find when out $K>1$, the system will be stable.

(b)

While,

$$K=1$$

The characteristic equation will become,

$$\Delta(s)=s^3+s^2+s+1=(s+1)(s^2+1)$$

The system have poles at $\pm j$, so the oscillation frequency will be $1\,\text{rad/sec}$

6.

A feedback control system has a characteristic equation:

$$s^3+(1+K)s^2+10s+(5+15K)=0$$

where the parameter K>0. Please use the Routh table to find:

(a) The value of K that the system will become unstable.

(b) The frequency of oscillation when K is the maximal value.

Solution

(a)

特性方程式

$$\Delta(s)=s^3+(1+K)s^2+10s+(5+15K)=0$$

$$\begin{array}{ccc} s^3 & 1 & 10 \\ s^2 & 1+K & 5+15K \\ s & \dfrac{5-5K}{1+K} & \end{array}$$

所以當 $K=1$時，系統將變成不穩定。

(b)

當 $K=1$時，令輔助方程式

$$A(s)=(1+K)s^2+(5+15K)=2s^2+20=0$$

得 $s=\pm j\sqrt{10}\ rad/\sec$

所以振盪頻率為 $\sqrt{10}\ rad/\sec$

7.

For the controlled plant $G(s)=\dfrac{s-1}{(s+4)(s-5)}$, please find a controller $C(s)$ such that the closed-loop transfer function of the unity feedback system is given by

$$G_f(s)=\frac{G(s)C(s)}{1+G(s)C(s)}=\frac{1}{s+5}$$

Is the closed-loop system asymptotically stable (why) ?

Solution:

$$G_f(s)=\frac{G(s)C(s)}{1+G(s)C(s)}=\frac{1}{s+5}=\frac{1}{s+5-1+1}=\frac{1}{1+(s+4)}$$

$$G(s)C(s)=\frac{1}{s+4}=\frac{s-1}{(s+4)(s-5)}\times C(s)$$

$$C(s)=\frac{s-5}{s-1}$$

發生右半平面極零點對消，故系統不穩定。