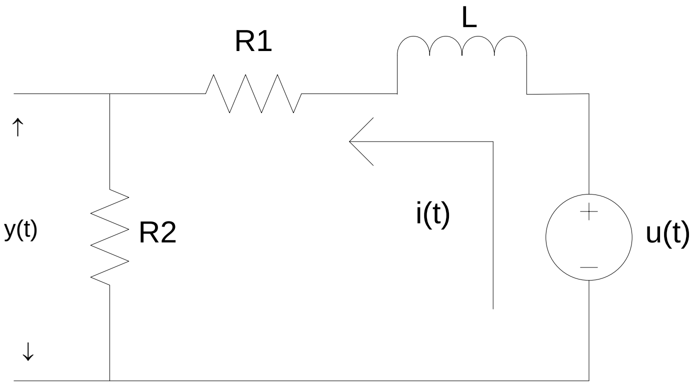
1.請求下圖電路轉移函數 $\frac{Y(s)}{U(s)}$ 之拉式轉換為何?



Solution：

克希何夫法:

由題目的中圖，可知克希何夫電壓迴路方程式為：

$$\left(\mathrm{R}_1+R_2\right)\times i\left(t\right)+L\frac{di(\mathrm{t})}{dt}=u\left(t\right) \qquad ...($$

令輸出電壓為

$$\mathrm{y}\left(\mathrm{t}\right)=R_2\times i\left(t\right) \qquad ...($$

由 $(1)\times\mathrm{R}_2$

$$\Rightarrow LR_2\frac{di(\mathrm{t})}{dt}+\left(\mathrm{R}_1+R_2\right)\times\left(R_2 i\left(t\right)\right)=R_2 u\left(t\right)$$

$$\mathrm{L}\frac{d\mathrm{y}\left(\mathrm{t}\right)}{dt}+\left(R_1+R_2\right)\mathrm{y}\left(\mathrm{t}\right)=R_2\times u\left(t\right)$$

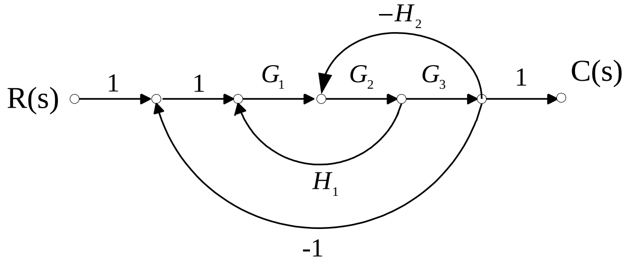
令初始條件為零，得上式之拉式轉換：

$$\mathrm{L}\times sY\left(\mathrm{s}\right)+\left(R_1+R_2\right)Y\left(\mathrm{s}\right)=R_2\times U\left(s\right)$$

因此求得轉移函數為：

$$\mathrm{G}\left(\mathrm{s}\right)=\frac{Y\left(\mathrm{s}\right)}{U\left(s\right)}=\frac{R_2}{Ls+(R_1+R_2)}=\frac{R_2}{R_1+R_2}\cdot\frac{1}{\frac{L}{R_1+R_2}s+1}$$

2. **一控制系統之信號流程圖如下圖所示，試用梅森法求** $\frac{C(s)}{R(s)}$ **。**



Solution：

在訊號流程圖中，

迴路增益有：$L_1 = G_1G_2H_1$, $L_2 = -G_2G_3H_2$, $L_3 = -G_1G_2G_3$

所有迴路均彼此接觸，因此沒有兩個以上未接觸的迴路增益乘積。

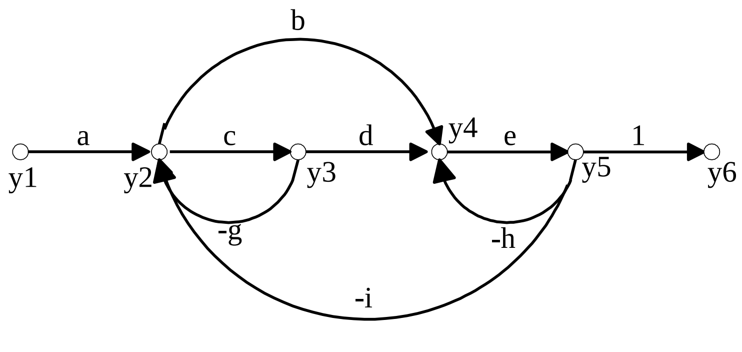
所有前進路徑增益 M_k 與相對應的 Δ_k 有

$M = G_1G_2G_3, \Delta_1 = 1 , \Delta = 1-(L_1 + L_2 + L_3)$

根據梅森增益公式：

$$\frac{C(s)}{R(s)} = \frac{\mathrm{M}}{\Delta} = \frac{G_1G_2G_3}{1-G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3}$$

3. **試用梅森法分別求下列訊號流程圖中，**$\frac{y_5}{y_1}, \frac{y_2}{y_1}, \frac{y_5}{y_2}$ **等的增益。**



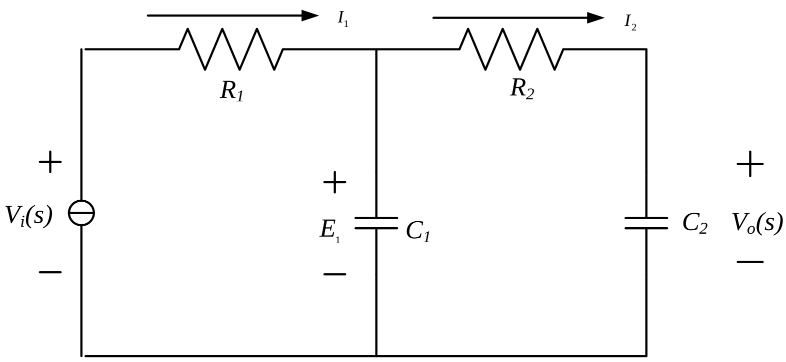
Solution：

$$\frac{y_5}{y_1}=\frac{acde+abe}{1+cg+eh+cdei+bei+cegh}$$

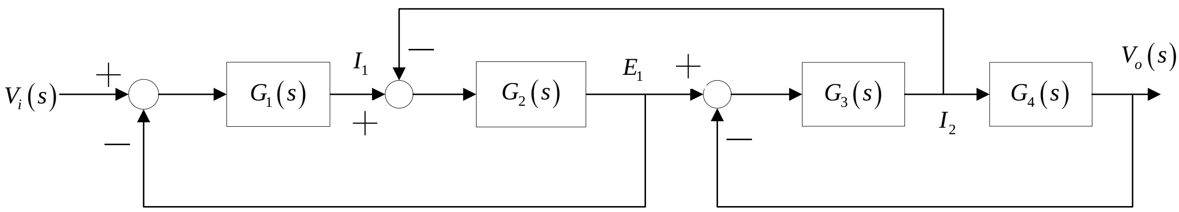
$$\frac{y_2}{y_1}=\frac{a(1+eh)}{1+cg+eh+cdei+bei+cegh}$$

$$\frac{y_5}{y_2}=\frac{\frac{y_5}{y_1}}{\frac{y_2}{y_1}}=\frac{cde+be}{1+eh}$$

4. **將圖(a)的電路改寫為如圖(b)的控制方塊圖表示**



圖(a)



圖(b)

請求出$G_1(s),G_2(s),G_3(s)$及$G_4(s)$**分別為何?**

Solution：

由$V_i - E_1 = I_1R_1$

可得

$$(I_1-I_2)\times\frac{1}{sC_1}=E_1$$

再由$(E_1-V_o)\times\frac{1}{R_2}=I_2$

及

$$I_2\times\frac{1}{sC_2}=V_o$$

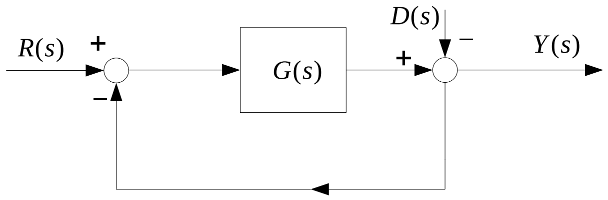
比較電路圖及方塊圖後可得：$G_1(s) = \frac{1}{R_1}$, $G_2(s) = \frac{1}{sC_1}$, $G_3(s) = \frac{1}{R_2}$, $G_4(s) = \frac{1}{sC_2}$

5.

令下圖中的閉迴路輸出 $Y(s) = T(s)R(s) - S(s)D(s)$ **，試推導**

(1) $S(s) = ?$

(2) $S(s) + T(s) = ?$



Solution：

(1)

$$Let\ D(s) = 0 \Rightarrow Y(s) = \frac{G(s)}{1+G(s)} R(s)$$

$$Let\ R(s) = 0 \Rightarrow Y(s) = \frac{-1}{1+G(s)} D(s)$$

$$\Rightarrow Y(s) = \frac{G(s)}{1+G(s)} R(s) - \frac{1}{1+G(s)} D(s)$$

比較得知 $T(s) = \frac{G(s)}{1+G(s)}$ & $S(s) = \frac{1}{1+G(s)}$

(2)

$S(s) + T(s) = 1$

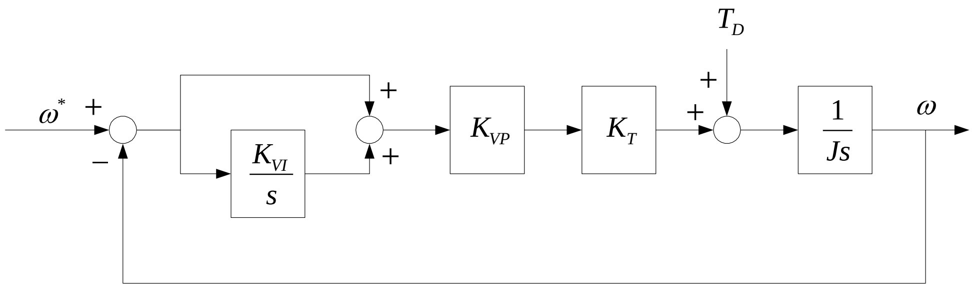
6. **針對下圖的馬達速度迴路系統，試推導**

(1) $T_{DIST}(s) = \frac{\omega(s)}{T_D(s)}$ **的轉移函數為何?**

(2) 高頻時，$T_{DIST}(s)$ **的振幅會近似於多少?**

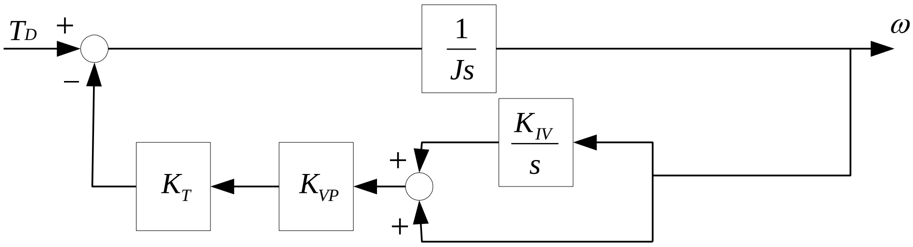
(3) 當 $\omega = 1\ rad/\sec$ **時，**$T_{DIST}(s)$ **的振幅會近似於多少?**

(4) 低頻時，$T_{DIST}(s)$ **的振幅會近似於多少?**



Solution：

重畫方塊圖



$$T_{Dist}=\frac{\frac{1}{Js}}{1+\frac{1}{Js}K_TK_{VP}(1+\frac{K_{IV}}{s})}=\frac{s}{Js^2+K_TK_{vp}s+K_TK_{vp}K_{vi}}$$

(2) $w\to\infty$ ，$T_{Dist}\to 0$

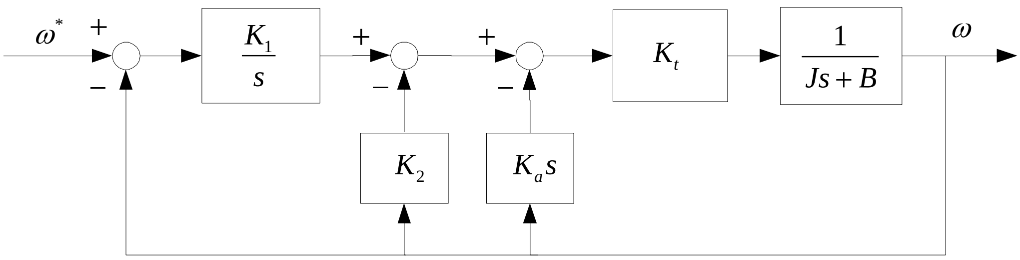
(3)

$$s=j\omega,\ \omega=1\Rightarrow T_{Dist}\to\frac{j}{(K_TK_{VP}K_{IV}-J)+K_TK_{VP}j}$$

$$|T_{Dist}|=\frac{1}{\sqrt{(K_TK_{VP}K_{IV}-J)^2+(K_TK_{VP})^2}}$$

(4) $w\to 0$ ，$T_{Dist}\to 0$

7. **請說明加速度回授的目的，並針對以下的系統方塊圖，推導出 $\frac{\omega}{\omega^*}$ 的轉移函數？**



Solution：

速度迴授的目的在於改變系統慣量，增加系統剛性，減少外擾對系統響應的影響

$$\frac{\omega}{\omega^*}=\frac{\dfrac{K_1K_t}{s(Js+B)}}{1+\dfrac{K_1K_t}{s(Js+B)}+\dfrac{K_tK_as}{Js+B}+\dfrac{K_tK_2}{Js+B}}$$

$$=\frac{K_1K_t}{(J+K_tK_a)s^2+(B+K_tK_2)s+K_1K_t}$$

8.

(a) Suppose a system $y=S(x)$ is linear, where y is output and x is input. Please describe the conditions of a linear system satisfies.

(b) Is $y=ax+b$ a linear system?

Solution

(a)

Assume

$$y_1=S(x_1)$$
$$y_2=S(x_2)$$

Let

$$x=\alpha_1x_1+\alpha_2x_2\Rightarrow y=S(x)=S(\alpha_1x_1+\alpha_2x_2)$$

If

$$y=\alpha_1S(x_1)+\alpha_2S(x_2)$$

which means

$$S(\alpha_1x_1+\alpha_2x_2)=\alpha_1S(x_1)+\alpha_2S(x_2)$$

We can say the system is linear.

(b)

Assume,

$$y = S(x) = ax + b$$
$$y_1 = S(x_1) = agx_1 + b$$
$$y_2 = S(x_2) = agx_2 + b$$

Let

$$y = S(x_1 + x_2) = a(x_1 + x_2) + b$$
$$S(x_1 + x_2) \neq S(x_1) + S(x_2)$$

The system does not satisfy the conditions of homogenous and superposition, which means the system is non-linear system.

9.

Let $G_1(\mathrm{s}) = \dfrac{1}{s+1}$ and $G_2(\mathrm{s}) = \dfrac{1}{s+2}$ represent the closed-loop transfer functions of two control systems.

(1) Which system has faster speed of response ? Why ?

(2) Which system has larger bandwidth ? Why ?

(3) Which system has smaller overshoot ? Why ?

Solution

(a)

$$G_1(\mathrm{s}) = \frac{1}{s+1} \quad \frac{\frac{1}{T}}{s+\frac{1}{T}} \Rightarrow T = 1\mathrm{sec}$$

$$G_2(\mathrm{s}) = \frac{1}{s+2} \quad \frac{k\frac{1}{T}}{s+\frac{1}{T}} \Rightarrow T = 0.5\mathrm{sec}$$

因此系統 $G_2(\mathrm{s})$ 響應較快，因為時間常數較短。

(b)

$G_1(\mathrm{s})$ 的頻寬

$$1 \times \frac{1}{\sqrt{2}} = \left| \frac{1}{j\omega + 1} \right| \Rightarrow \omega = 1 \, {}^{rad}\!/_{\mathrm{sec}}$$

$G_2(\mathrm{s})$ 的頻寬

$$\frac{1}{2} \times \frac{1}{\sqrt{2}} = \left| \frac{1}{j\omega + 2} \right| \Rightarrow \omega = 2 \, {}^{rad}\!/_{\mathrm{sec}}$$

因此系統$G_2(s)$有較大的頻寬。

(c)

兩系統皆為一階，故無超越量。