電子學

1.4

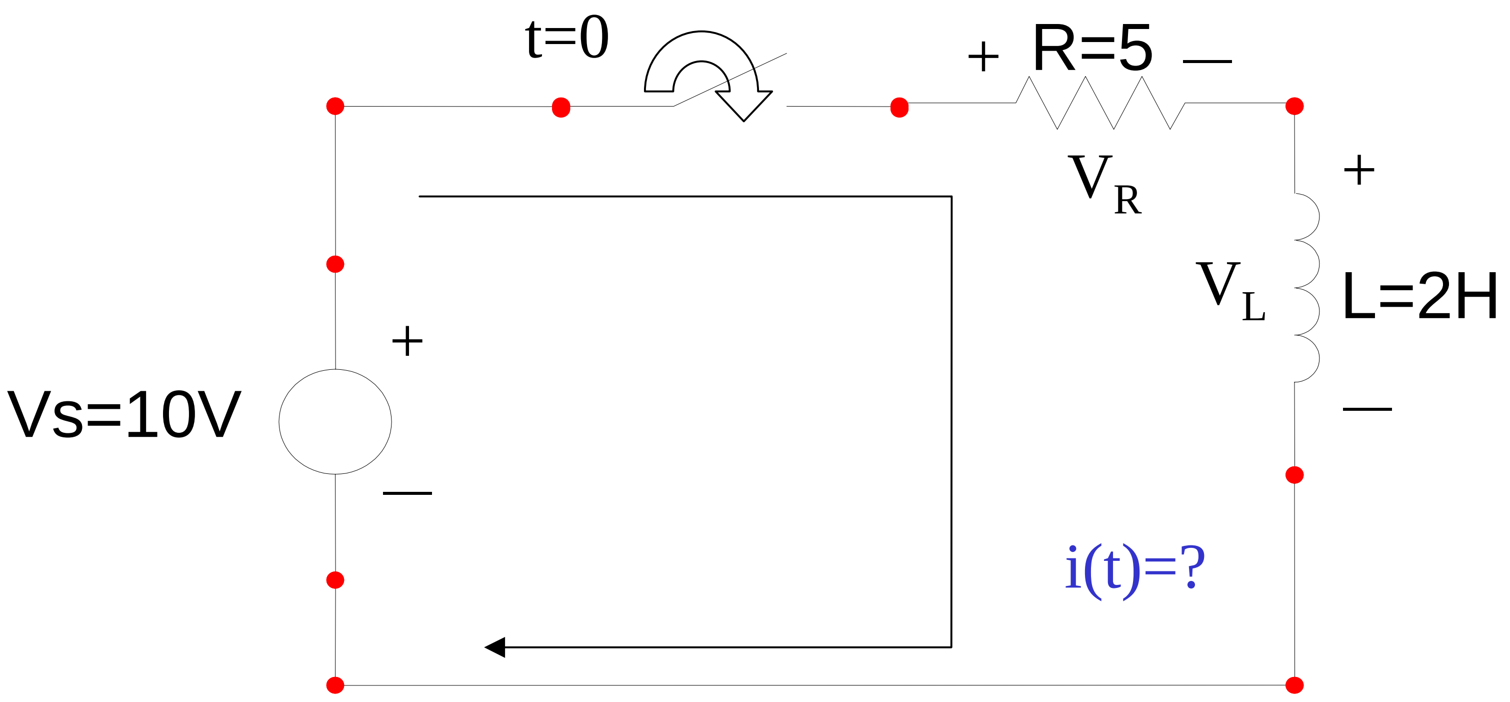
DC TRANSIENT ANALYSIS

Objective

- **We will discuss the response of first-order RL or RC circuits to sudden changes in the circuit, usually as the result of some switch action.**

- **Two methods are presented here for solving first-order transient problems :**
  - **Write and solve the different equation.**
  - **Derive solutions based on initial values, final values, and the time constant.**

Deriving the differential equation



An RL transient problem. The switch is closed and a current builds up in the circuit. We will solve for the current after the switch is closed.

After the instant at which the switch is closed,

From KVL for the circuit：

$$-V_S+V_R(t)+V_L(t)=0 \quad (1.79)$$

$$L\frac{di(t)}{dt}+Ri(t)=V_s,\, t>0 \quad (1.80)$$

The general solution to Eq (1.80)

$$i(t)=\mathbf{A}+\mathbf{B}e^{\alpha t} \quad (1.81)$$

Substiuting Eq.(1.81) back into Eq(1.80)

$$\boldsymbol{LB}\,\alpha e^{\alpha t}+R(A+Be^{\alpha t})=V_S \quad (1.82)$$

$$\mathbf{B}(\mathbf{L}\,\alpha+R)e^{\alpha t}+AR=V_S$$

In the form $e^{-t/\tau}$

$$\tau = -1/\alpha = L/R \text{ seconds} \qquad (1.84)$$

The coefficient of the exponential term must vanish if the equation is valid for all times

$$L\alpha + R = 0$$

$$\Rightarrow \quad \alpha = -R/L = -2.5 s^{-1} \qquad (1.83)$$

The time constant τ

It is a characteristic time for the transient ; we will explore its significance shortly.

Setting the coefficient of the exponential term to zero leads also to the value of A, the steady-state response

$$AR=V_S$$

$$\Rightarrow A=V_S/R=10/5=2A$$

Because i(0^+)=0

$$0=A+Be^{-0/\tau} \qquad B=-A=-2 \text{ amperes}$$

The finial solution

$$i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{-t/0.4s} \text{ A} \qquad (1.87)$$

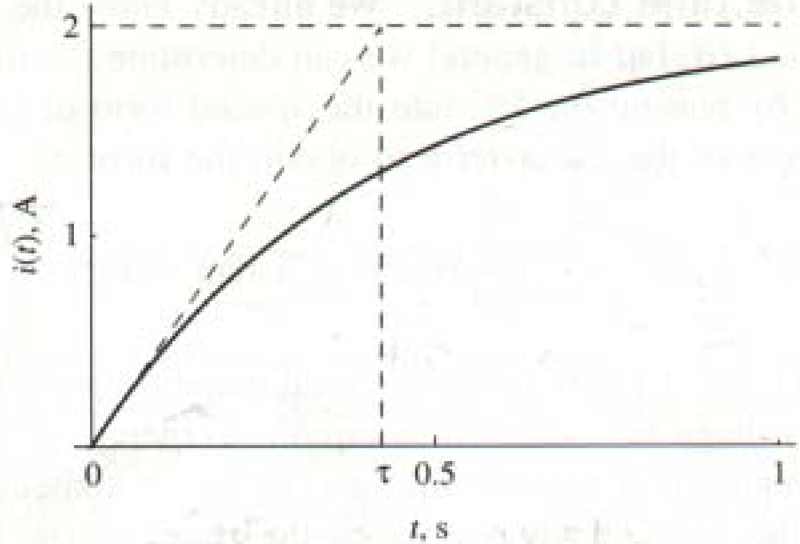


Figure 1.42 Current for circuit in Fig. 1.41.

Initial and Final Values

By initial we mean the instant after a change occurs in the circuit , usually a switch closing or opening.

By final we mean the steady-state condition of the circuit,its state After a large period of time.

Initial Values

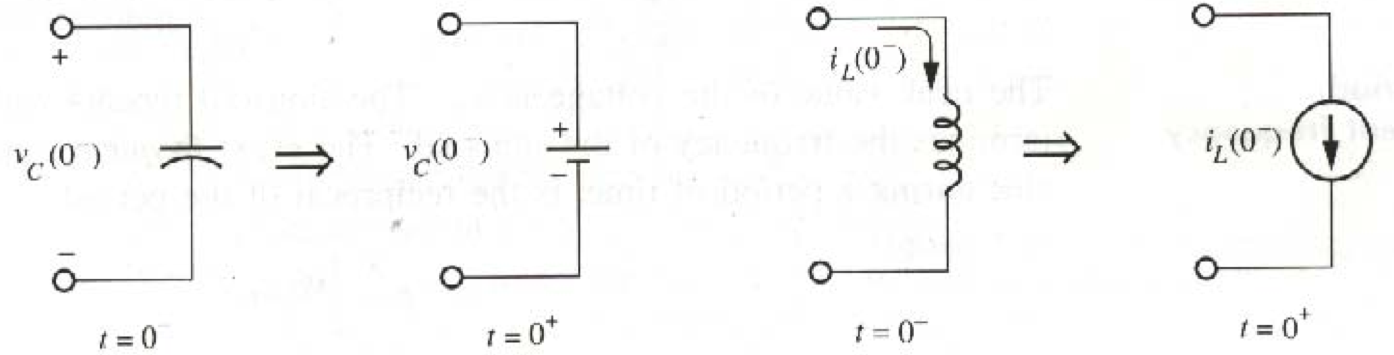


Figure 1.47 A battery models the initial voltage of a capacitor.

Figure 1.48 A current source models the initial current of an inductor.

In Fig.1.47,a charged capacitor acts like a voltage source because the voltage across the capacitor cannot change instantaneously.

Similarly in Fig 1.48 a current source models and energized inductor at the instant after switch action.

Final Values

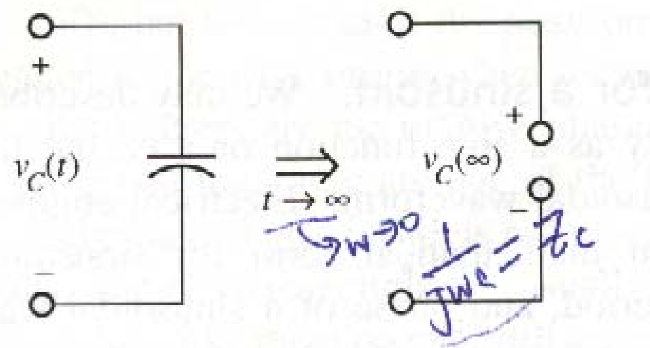


Figure 1.45 The capacitor behaves as an open circuit as time becomes large.

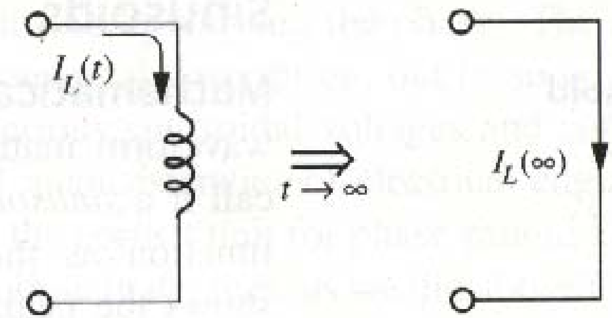


Figure 1.46 The inductor behaves as a short circuit as time becomes large.

The current through capacitors and the voltage across inductors must approach zero as suggested in Eq(19.92)

$V_C \to$ constant => $i_c = c\dfrac{dv_c}{dt}$ → 0 as $t \to 0$

$i_L \to$ constant => $v_L = L\dfrac{di_L}{dt}$ → 0 as $t \to 0$