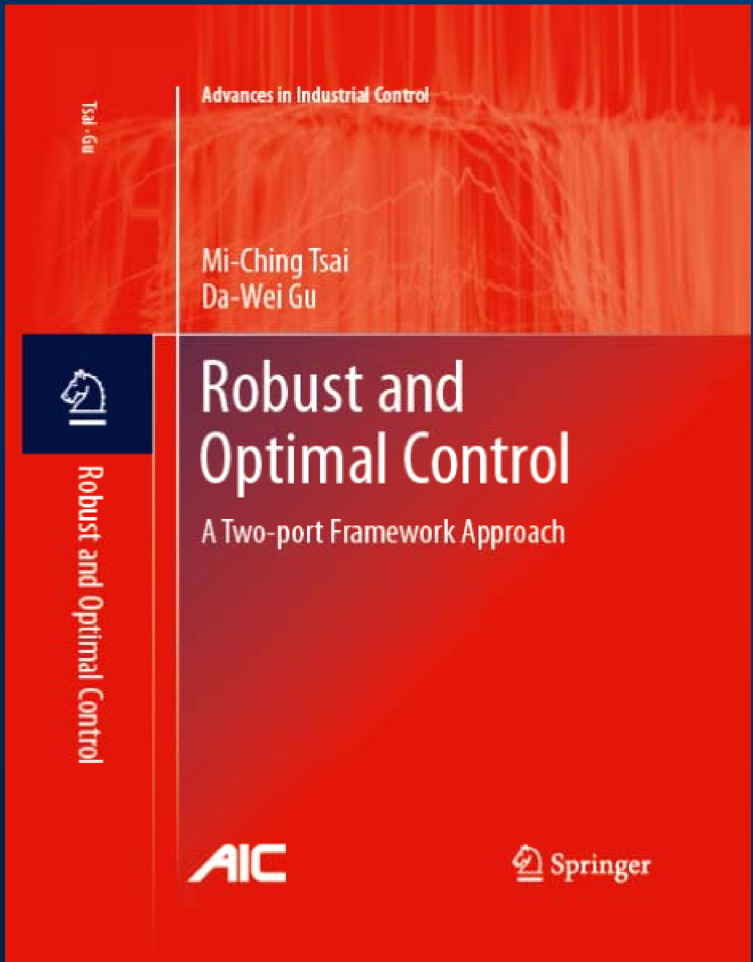


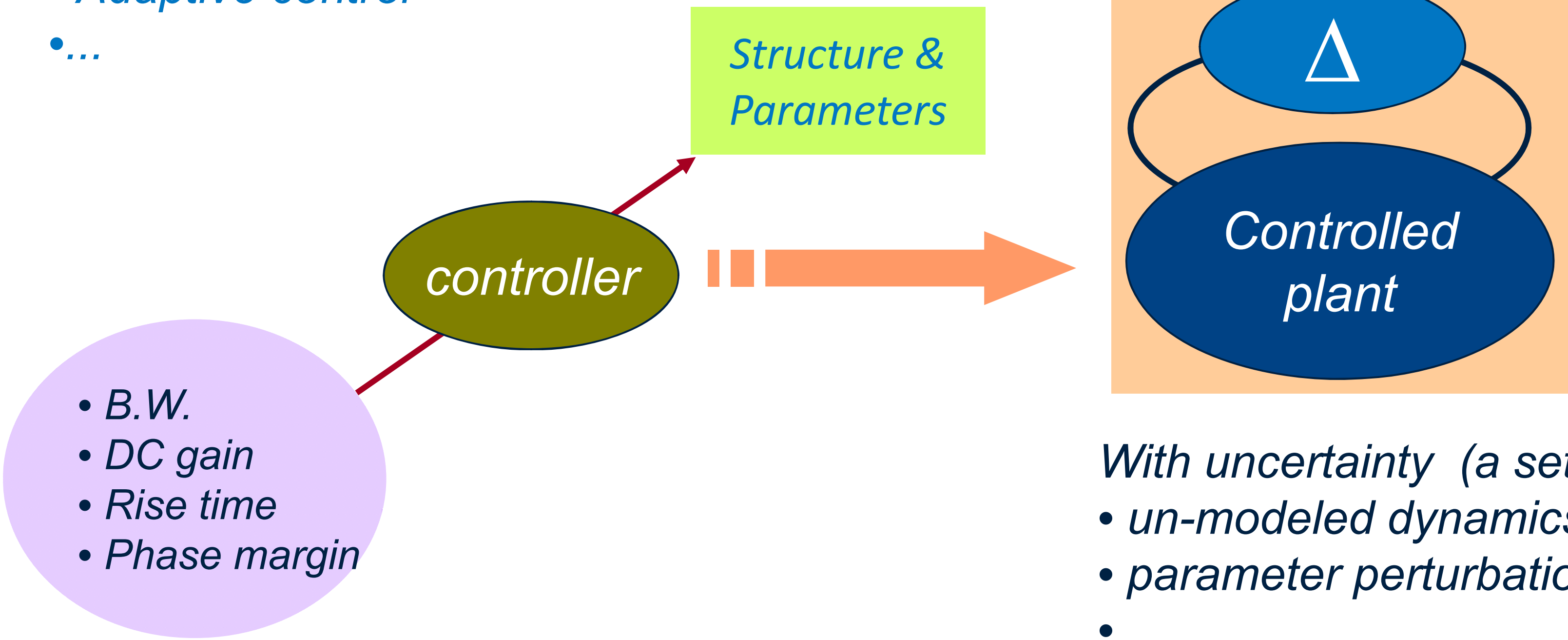
Robust and Optimal Control

A Two-port Framework Approach

CSD approach to H-infinity Controllers

Control Design

- Robust control
- Adaptive control
- ...

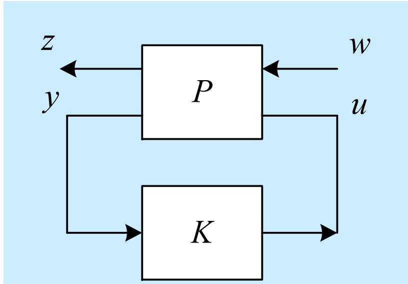


With uncertainty (a set)
- *un-modeled dynamics*
- *parameter perturbation*
- ...

- *Stabilized controller: to make the closed-loop poles stable*
- *H_2 control: to minimize the 2-norm* $\|G\|_2^2 = \|LFT_l(P,K)\|_2^2$
- *H_∞ control: to minimize the infinity-norm* $\|G\|_\infty = \sup_\omega \bar{\sigma}\left(G(j\omega)\right)$

Classical approach to H-infinity control

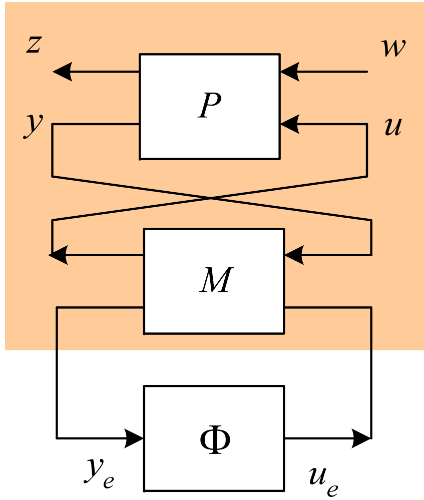
LFT



Find all stabilizing solutions

Model matching Problem

$$\min_{K \in RL_\infty} \left\| LFT_l(P,K) \right\|_\infty = \min_{\Phi \in RH_\infty} \left\| T_{11} + T_{12}\Phi T_{21} \right\|_\infty$$



T

Inner-outer factorization

1, 2 or 4-block problem

$$\min_{\Phi \in RH_\infty} \left\| \begin{matrix} R_{11} + \Phi & R_{12} \\ R_{21} & R_{22} \end{matrix} \right\|_\infty$$

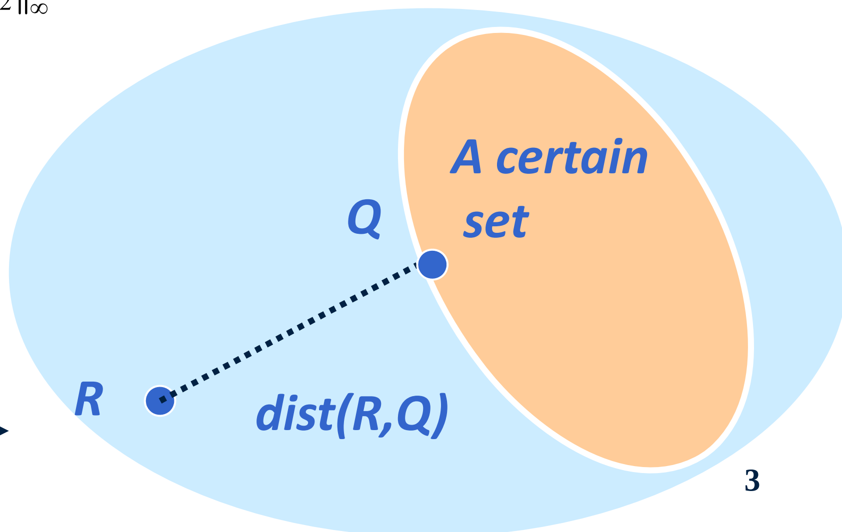
Spectral factorization

Nehari problem

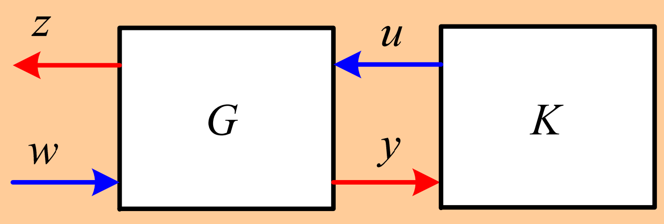
$$\min_{Q \in RH_\infty} \left\| R - Q \right\|_\infty$$

Hankel Approximation

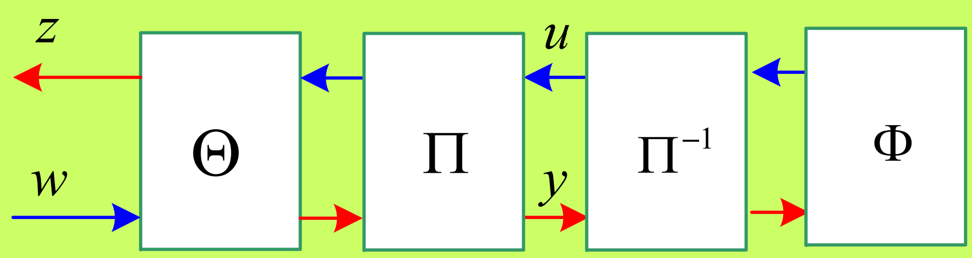
Find (sub-) optimal Q



A CSD approach is brought up by Kimura, (1991~1996)

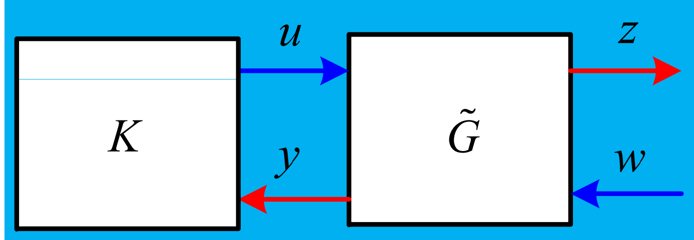


$$G = \begin{bmatrix} P_{12} - P_{11}P_{21}^{-1}P_{22} & P_{11}P_{21}^{-1} \\ -P_{21}^{-1}P_{22} & P_{21}^{-1} \end{bmatrix}$$

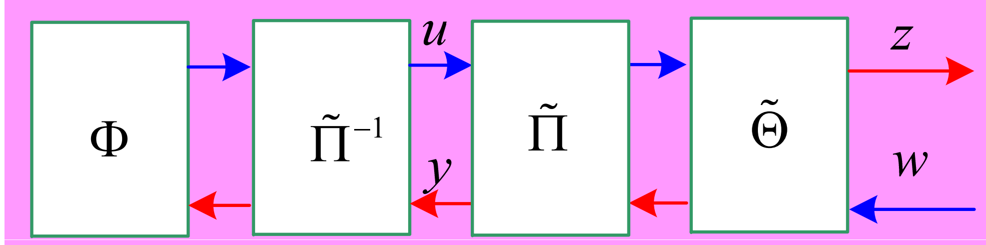


$$G = \Theta\Pi \qquad K = CSD_r(\Pi^{-1}, \Phi)$$

If Θ is J-lossless $\left\| CSD_r(G,K) \right\|_\infty = \left\| CSD_r(\Theta,\Phi) \right\|_\infty < 1 \quad \forall \ \|\Phi\|_\infty < 1$



$$G = \begin{bmatrix} P_{12}^{-1} & -P_{12}^{-1}P_{11} \\ P_{22}P_{12}^{-1} & P_{21} - P_{22}P_{12}^{-1}P_{11} \end{bmatrix}$$

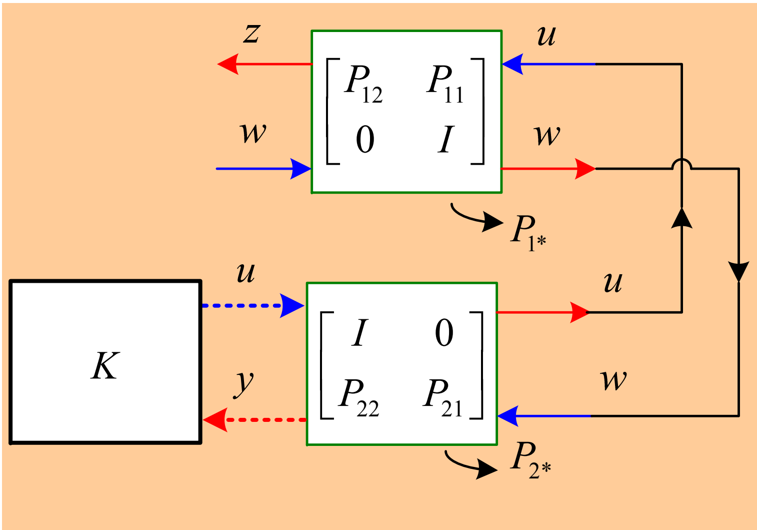


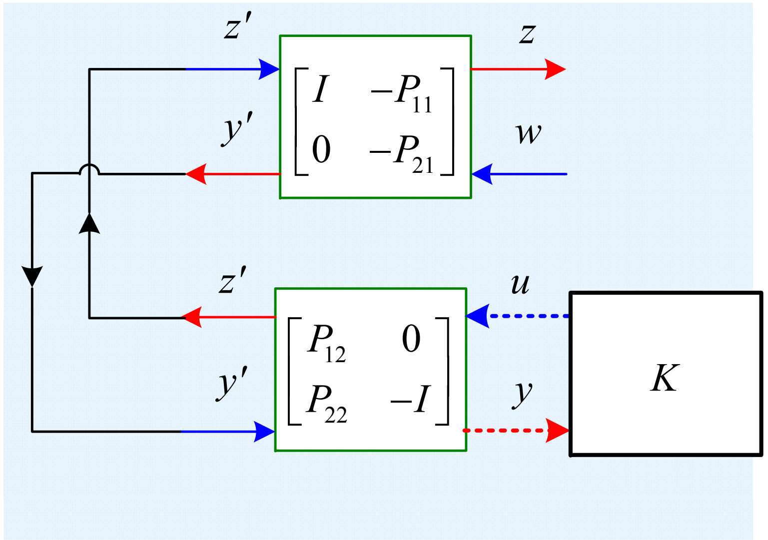
$$G = \tilde{\Pi}\Theta \qquad K = CSD_l(\tilde{\Pi}^{-1}, \Phi)$$

If $\tilde{\Theta}$ is dual J-lossless $\left\| CSD_l(G,K) \right\|_\infty = \left\| CSD_l(\tilde{\Theta},\Phi) \right\|_\infty < 1 \quad \forall \ \|\Phi\|_\infty < 1$

Augmentation is needed if P_{12} or P_{21} is not invertible

The proposed coupled CSD method





Advantages:

(1) These two topologies are always exist neither P_{12} nor P_{21} is not invertible.

(2) The coprime factorization can be easily achieved by multiplying a matrix in the right/left side terminal.