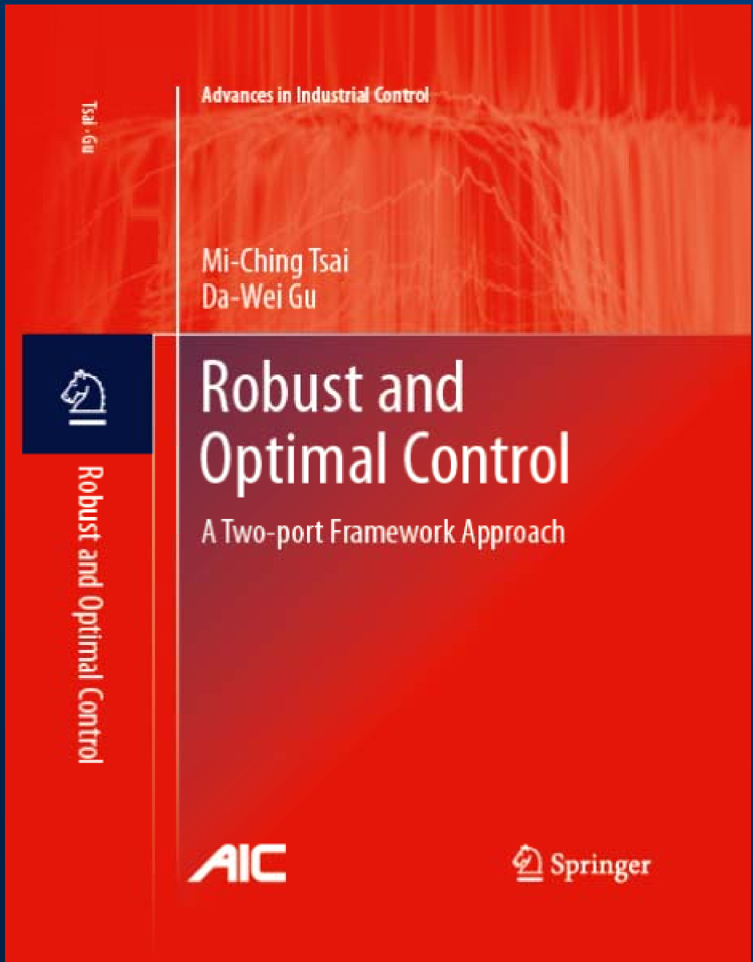


# Robust and Optimal Control

## A Two-port Framework Approach

# Algebraic Riccati Equations and Spectral Factorizations

# *Algebraic Riccati Equations (ARE)*

## Algebraic Riccati Equations (ARE)

- ***Definition***
  - $A$, $R$, $Q$ : n*n real coefficient matrices
  - $R$, $Q$ : symmetric

$$A^T X + XA + XRX + Q = 0$$

where

$$R = R^T$$

$$Q = Q^T \geq 0$$

## *Hamiltonian matrix*

- ***ARE:*** $$A^T X + XA + XRX + Q = 0$$

- ***Define*** $$H := \begin{bmatrix} A & R \\ -Q & -A^T \end{bmatrix} \in R^{2n \times 2n}$$

Hamiltonian Matrix

Satisfied

$$J^{-1} H J = -H^T$$

where $J = \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix}$ , $J^2 = -I$ and $J^T = -J = J^{-1}$

## Definition of Notations

- *Ric* : $R^{2n\times 2n} \rightarrow R^{n\times n}$
- *X=Ric(H)*
  - $H$ : Hamiltonian matrix.
  - $X$ : A solution to the ARE given by $H$
  - $X$ makes $A+RX$ Hurwitz.
- $H \in dom(Ric)$
  - H does not have eigenvalues on the imaginary axis