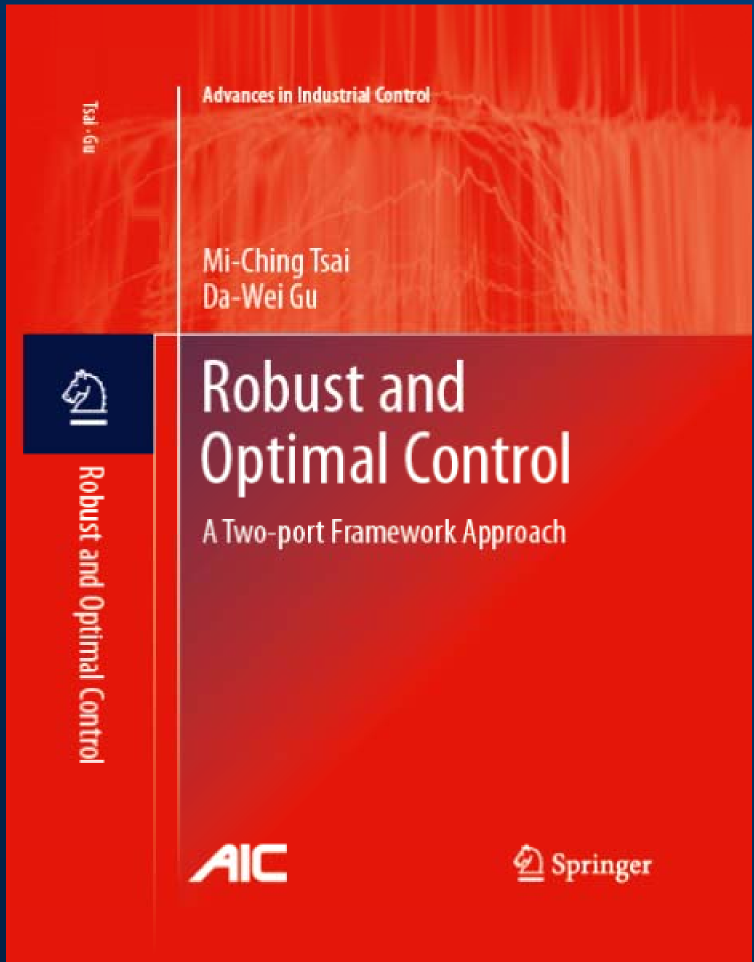


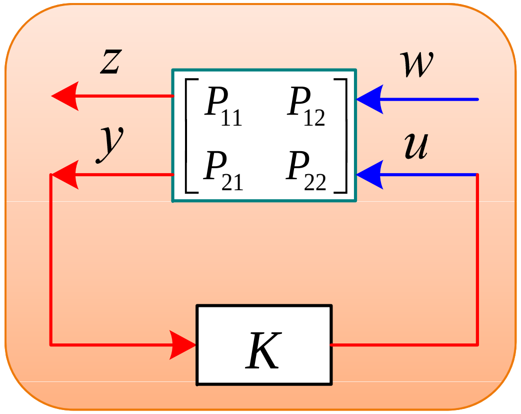
Robust and Optimal Control

A Two-port Framework Approach

Chain Scattering Descriptions

A Standard Formulation for General Control Problems (LFT_l)

Consider a general control problem



$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \qquad u = Ky$$

$$\begin{bmatrix} z \\ y \end{bmatrix} = P \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}$$

$$P_{11} = \frac{z}{w}\Big|_{u=0} \qquad P_{12} = \frac{z}{u}\Big|_{w=0}$$

$$P_{21} = \frac{y}{w}\Big|_{u=0} \qquad P_{22} = \frac{y}{u}\Big|_{w=0}$$

$$\Rightarrow z = P_{11}w + P_{12}u$$

$$y = P_{21}w + P_{22}u = P_{21}w + P_{22}Ky$$

$$\Rightarrow (I - P_{22}K)\,y = P_{21}w$$

$$\Rightarrow y = (I - P_{22}K)^{-1} P_{21}w$$

$$z = P_{11}w + P_{12}u = P_{11}w + P_{12}K(I - P_{22}K)^{-1} P_{21}w$$

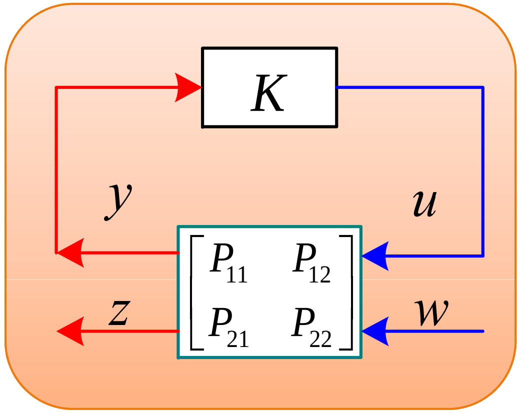
$$= \left[P_{11} + P_{12}K(I - P_{22}K)^{-1} P_{21}\right] w$$

$$= LFT_l(P, K)w$$

$$LFT_l(P, K) := P_{11} + P_{12}K(I - P_{22}K)^{-1} P_{21}$$

Another Formulation for General Control Problems (LFT_u)

Consider the general control problem



$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \qquad u = Ky$$

$$\begin{bmatrix} y \\ z \end{bmatrix} = P \begin{bmatrix} u \\ w \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} u \\ w \end{bmatrix}$$

$$P_{11} = \frac{y}{u}\Big|_{w=0} \qquad P_{12} = \frac{y}{w}\Big|_{u=0}$$

$$P_{21} = \frac{z}{u}\Big|_{w=0} \qquad P_{22} = \frac{z}{w}\Big|_{u=0}$$

$$\Rightarrow z = P_{21}u + P_{22}w$$

$$y = P_{11}u + P_{12}w = P_{11}Ky + P_{12}w$$

$$\Rightarrow (I - P_{11}K)\, y = P_{12}w$$

$$\Rightarrow y = (I - P_{11}K)^{-1} P_{12}w$$

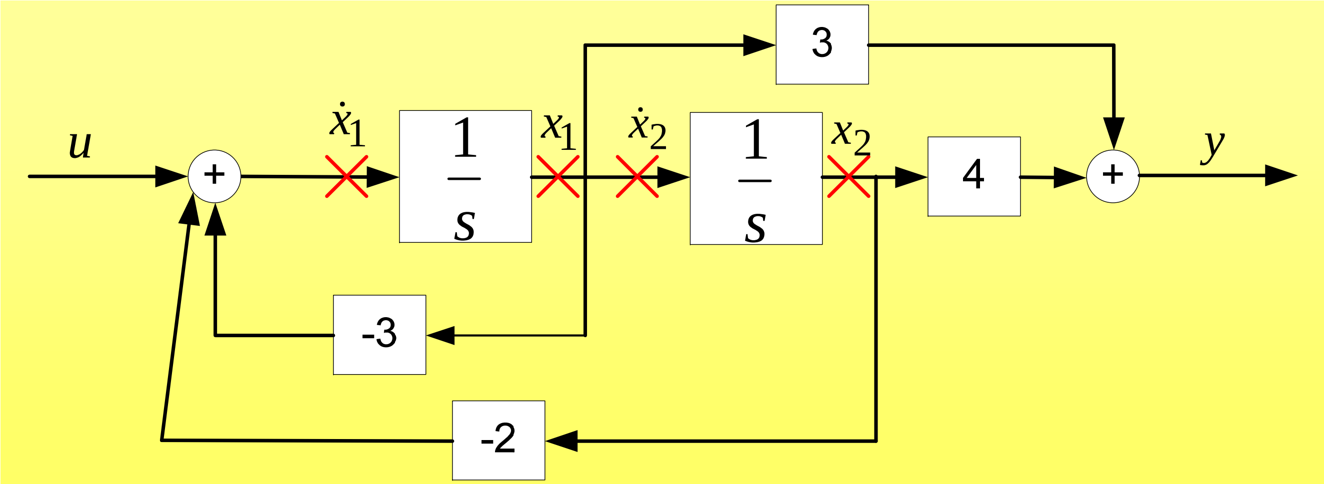
$$z = P_{21}u + P_{22}w = P_{21}K(I - P_{11}K)^{-1} P_{12}w + P_{22}w$$

$$= \left[P_{21}K(I - P_{11}K)^{-1} P_{12} + P_{22} \right] w$$

$$= LFT_u(P, K)w$$

$$LFT_u(P, K) := P_{22} + P_{21}K(I - P_{11}K)^{-1} P_{12}$$

An Example: State Space Realization (LFT_u)

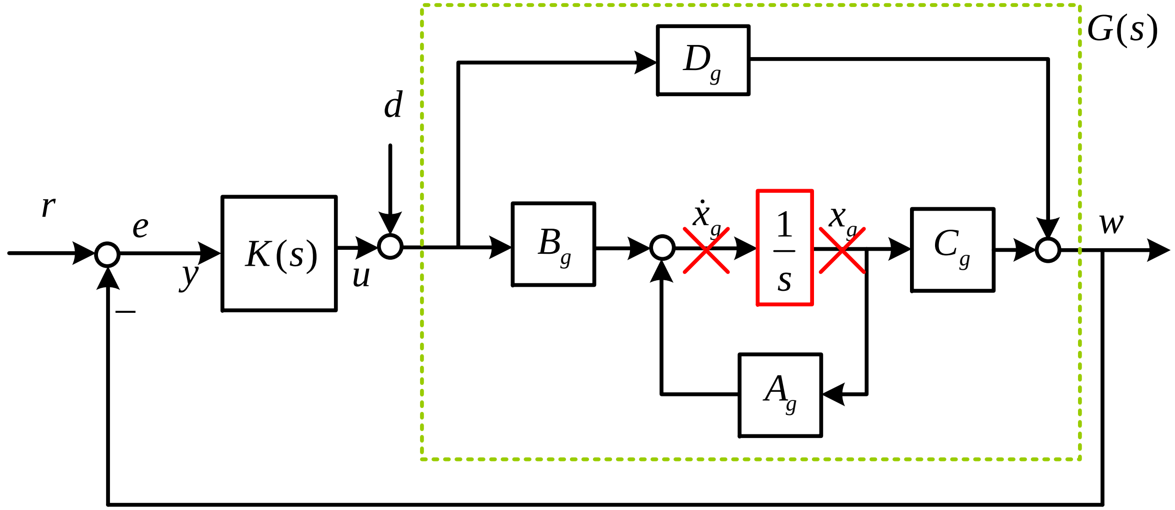


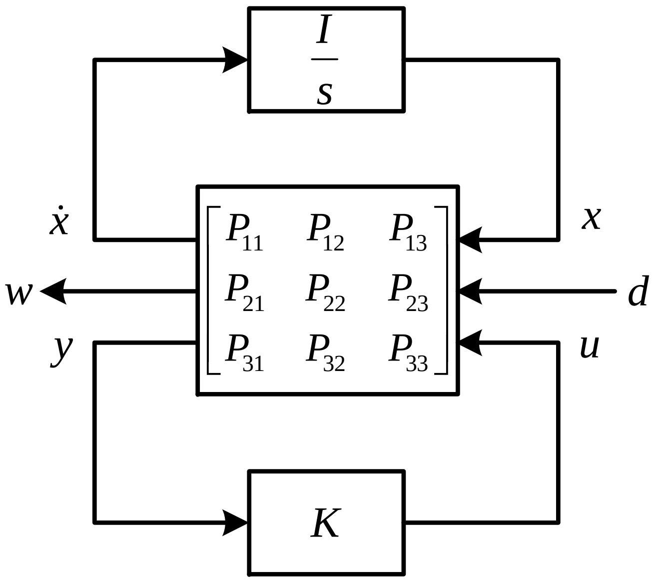
$$\begin{array}{c} \\ \dot{x}_1 \\ \dot{x}_2 \\ y \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & u \end{array} \\ \left[\begin{array}{cc|c} -3 & -2 & 1 \\ 1 & 0 & 0 \\ \hline 3 & 4 & 0 \end{array}\right] \end{array} = \left[\begin{array}{c|c} P_{11} & P_{12} \\ \hline P_{21} & P_{22} \end{array}\right] = \left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array}\right] \qquad K = \begin{bmatrix} 1/s & 0 \\ 0 & 1/s \end{bmatrix} \iff \begin{array}{l} \dot{x} = Ax + Bu \\ y = Cx + Du \end{array}$$

$$LFT_u\left(P, \frac{I}{s}\right) = P_{22} + P_{21}\frac{I}{s}\left(I - \frac{P_{11}}{s}\right)^{-1} P_{12} = 0 + \begin{bmatrix} 3 & 4 \end{bmatrix} \frac{I}{s}\left(I - \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} \frac{I}{s}\right)^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$LFT_u\left(P, \frac{I}{s}\right) = \frac{3s+4}{s^2+3s+2}$$

A Feedback System Representation (*LFT*)





$$\begin{bmatrix} \dot{x} \\ w \\ y \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} \begin{bmatrix} x \\ d \\ u \end{bmatrix}$$

$$= \begin{bmatrix} A_g & B_g & B_g \\ C_g & D_g & D_g \\ -C_g & -D_g & -D_g \end{bmatrix} \begin{bmatrix} x \\ d \\ u \end{bmatrix}$$