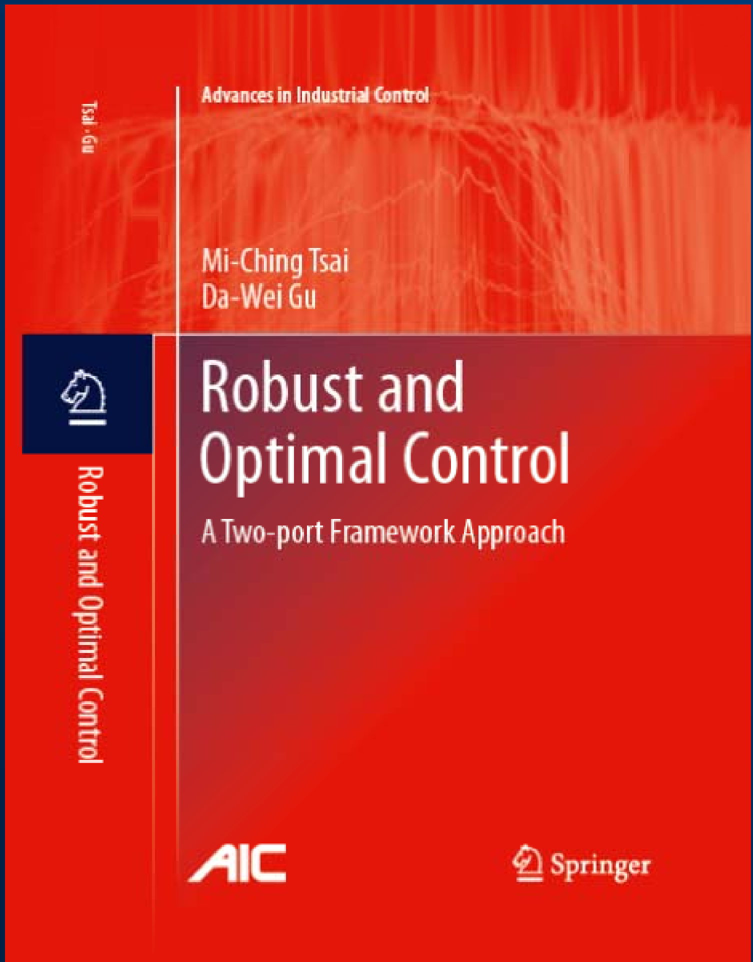


Robust and Optimal Control

A Two-port Framework Approach

Linear Fractional Transformation

Definition of a linear system

If f(x) is linear, then it satisfies

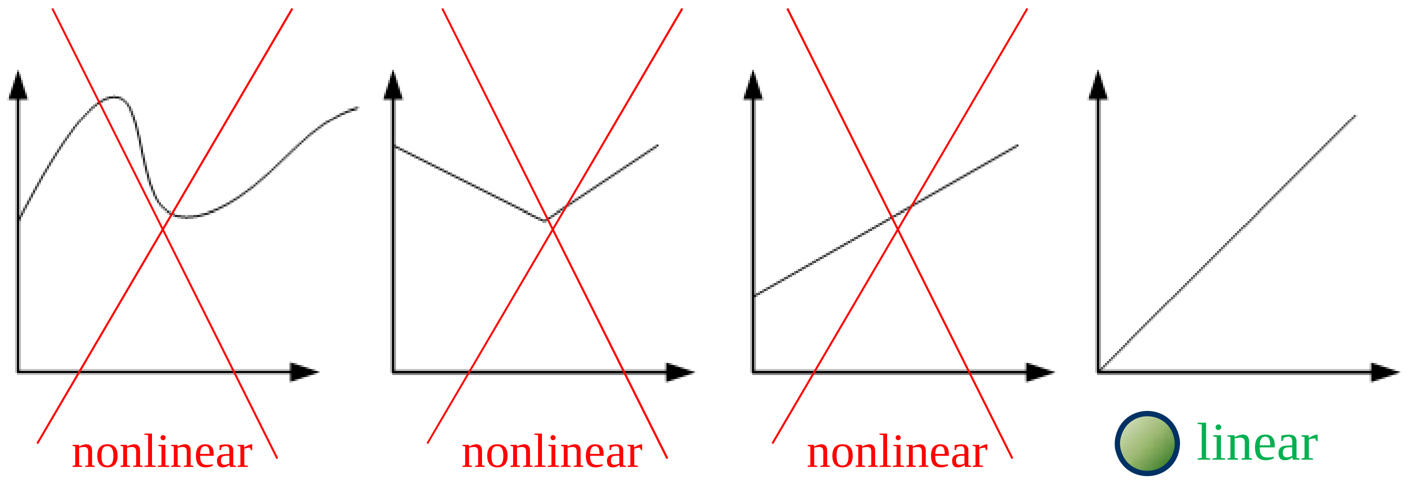
1. homogenous

   $f(ax) = af(x)$

2. superposition

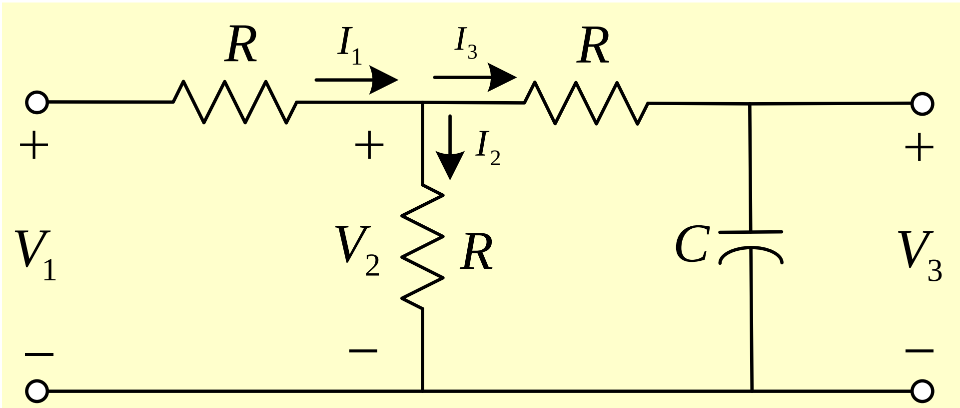
   $f(x_1 + x_2) = f(x_1) + f(x_2)$ or

   $f(a_1 x_1 + a_2 x_2) = a_1 f(x_1) + a_2 f(x_2)$



Transfer Function

Example: Determine the transfer function of the following circuit.



<u>Method 1:</u>

$$\frac{V_3}{V_1} = K\frac{\tau_Z s + 1}{\tau_P s + 1}$$

- $s \to 0$ → K
- $V_3 = 0$ → τ_Z
- $V_1 = 0$ → τ_P

<Physical Insight>

$$\begin{cases} s \to 0 : \text{the time goes to infinity} \\ V_3 = 0 : \text{the short circuit in output} \\ V_1 = 0 : \text{the short circuit in input} \end{cases}$$

Transfer Function

$a)$ time goes to infinity $s \to 0$: *dc gain*

The capacitor is open circuit. $\frac{V_3}{V_1} = \frac{R}{R+R} = \frac{1}{2}$

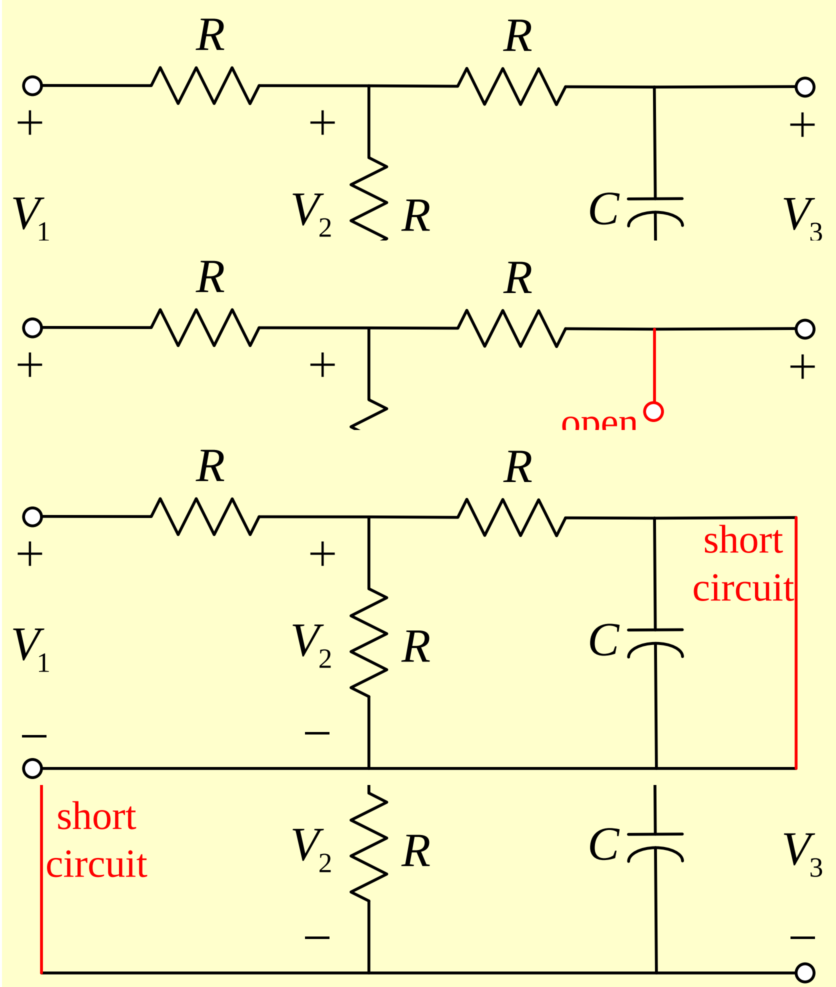
$b) V_3 = 0$ the output is short circuit

τ_z : the time constant of the existing circuit

$$\tau_z = R_{eq} \times C_{eq} = 0 \times C_{eq} = 0$$

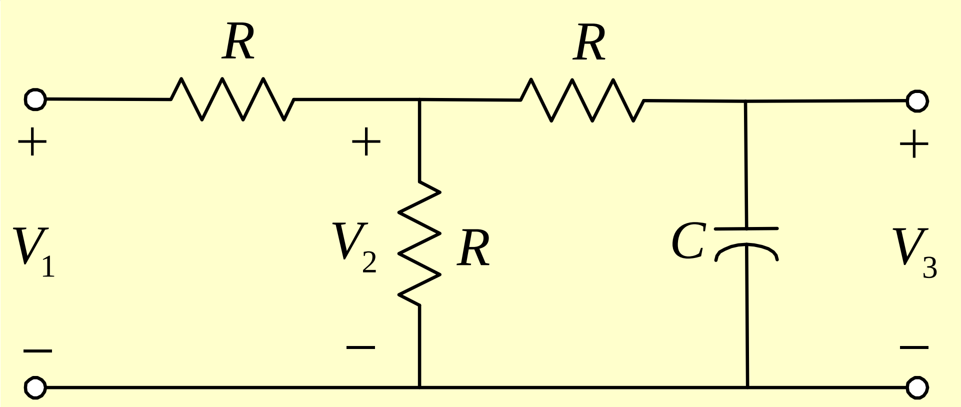
$c) V_1 = 0$ the input is short circuit

$$\tau_p = R_{eq} \times C_{eq} = \frac{3}{2} R \times C$$



$$T(s) = \frac{1}{2} \frac{1}{(\frac{3RC}{2})s + 1} = \frac{1}{3RCs + 2}$$

Example:



Method 2: Mason's Rule

$$M = \frac{\sum_j M_j \Delta_j}{\Delta}$$

M = transfer function or gain of the system

M_j = gain of one forward path

j = an integer representing the forward paths in the system

Δ_j = 1+the loops remaining after removing path j.

(1+the loops that don't touch the forward path M_j)

If none remain, then $\Delta_j = 1$

$\Delta = 1 - \sum$ loop gains $+ \sum$ nontouching loop gains taken two at a time

$- \sum$ nontouching loop gains taken three at a time

$+ \sum$ nontouching loop gains taken four at a time $\cdots$