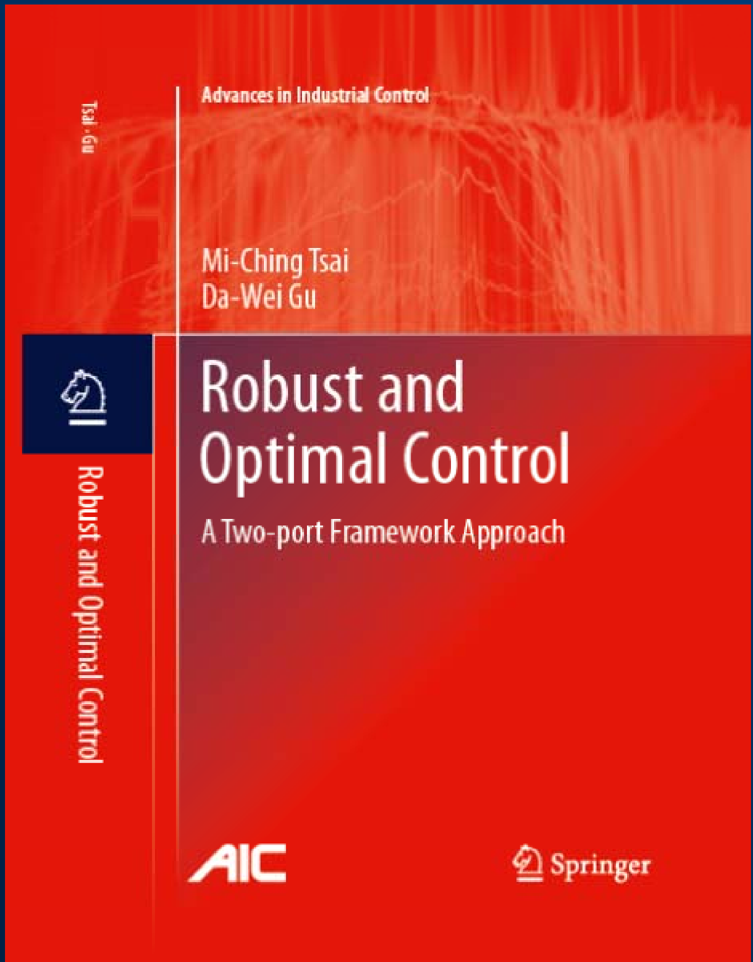
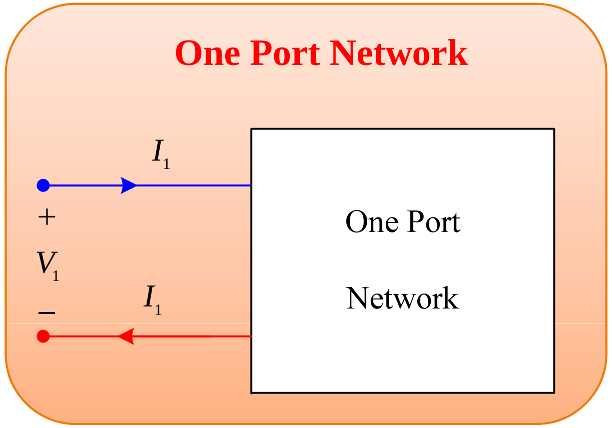


Robust and Optimal Control

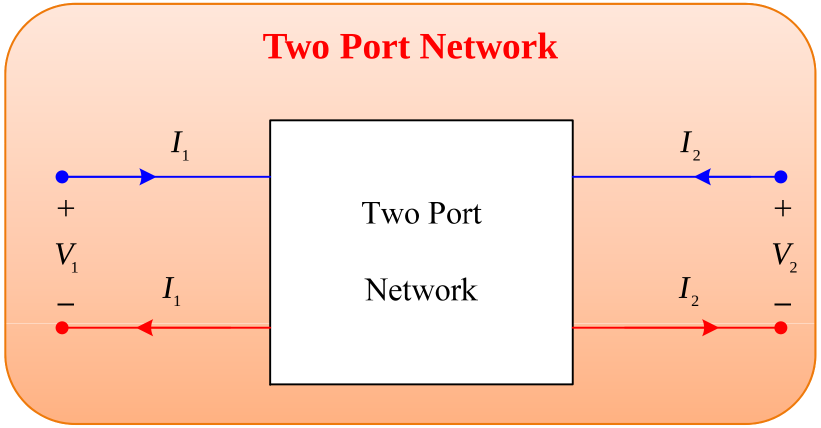
A Two-port Framework Approach

Two-port Networks

Introduction to two port network



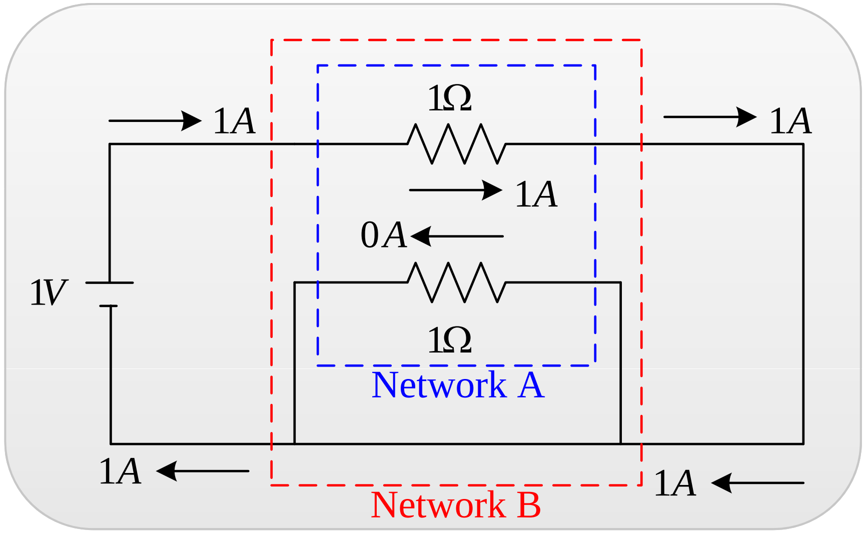
Definition: $I_1(input) = I_1(output)$



Definition: $I_1(input) = I_1(output)$

$I_2(input) = I_2(output)$

Example: Which circuit is two port network?



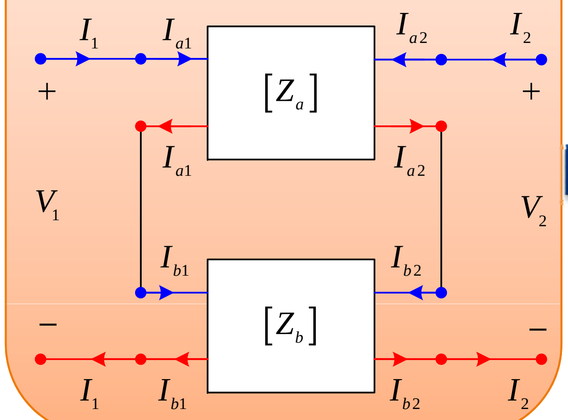
Ans.: Network A is't，Network B is.

Connection of Two Port Network

- Series connection
- Parallel connection
- Cascade connection

Two ports network may be interconnected to create more complex network:

(1) Series connection

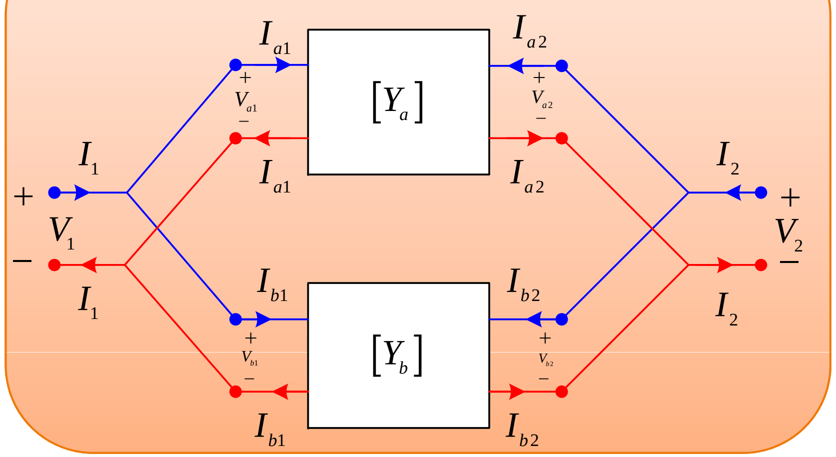


Z : Impedance matrix

$Z_{sum} = Z_a + Z_b$

$$\begin{cases} I_1 = I_{a1} = I_{b1} \\ I_2 = I_{a2} = I_{b2} \end{cases}$$

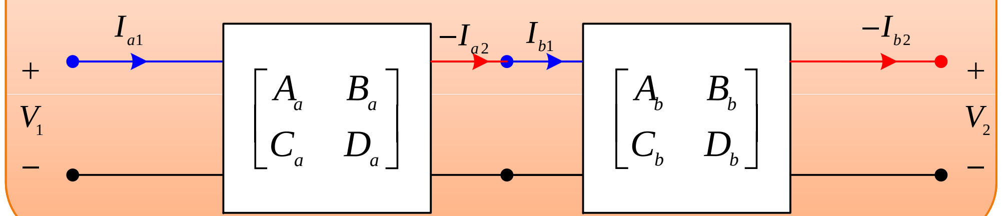
(2) Parallel connection (Shunt connection)



Y : Admittance matrix, $Y_{sum} = Y_a + Y_b$

$$\begin{cases} V_{a1} = V_{b1} = V_1 \\ V_{a2} = V_{b2} = V_2 \end{cases} \qquad \begin{cases} I_1 = I_{a1} + I_{b1} \\ I_2 = I_{a2} + I_{b2} \end{cases}$$

(3) Cascade connection (Chain)



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}, \quad -I_{a2} = I_{b1}$$

Comparison of Electrical and Mechanical Impedance

Mechanical		Electrical	
Force	F	Voltage	V
Displacement	x	Charge	q
Velocity	v	Current	i
Mass	M	Inductance	L
Damper	B	Resistance	R
Spring	K	Capacitance	$\frac{1}{C}$
Power	$F \times v$	Power	$V \times i$

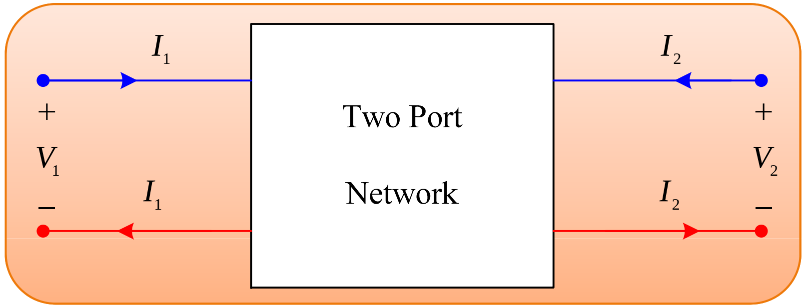
Force-Voltage Analogies

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = V$$

$$m\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx = F$$

Parameters of Two Port Network

Impedance Parameters (Z Parameters)

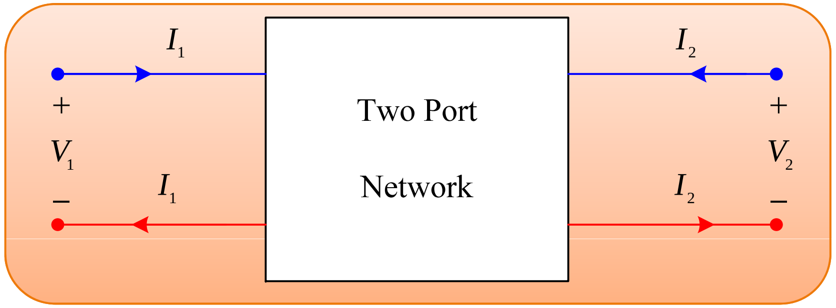


$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$
, therefore

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \qquad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

Admittance Parameters (Y Parameters)



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$
, therefore

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} \qquad Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$