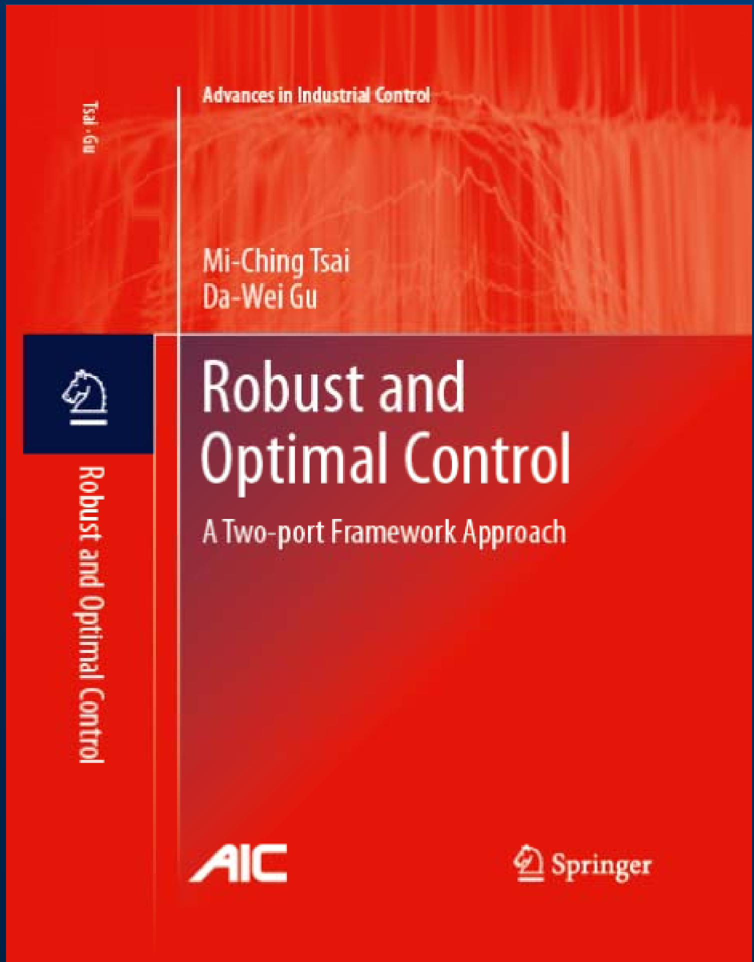


Robust and Optimal Control

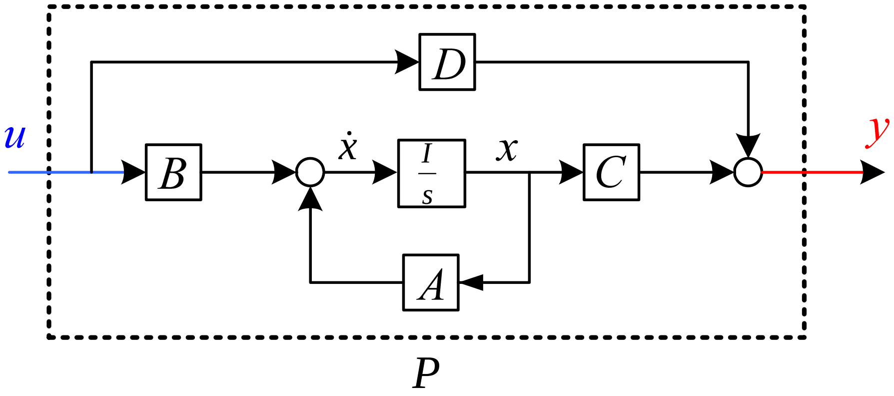
A Two-port Framework Approach

Some Basic Concepts

Some Useful Mathematic Tools

State-space formula of a system

Let $y = P(s)\cdot u$ with $P(s) \overset{s}{=} \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ shown below



Note that $P(s) \overset{s}{=} \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ **denotes a system instead of a matrix here, since you can't**

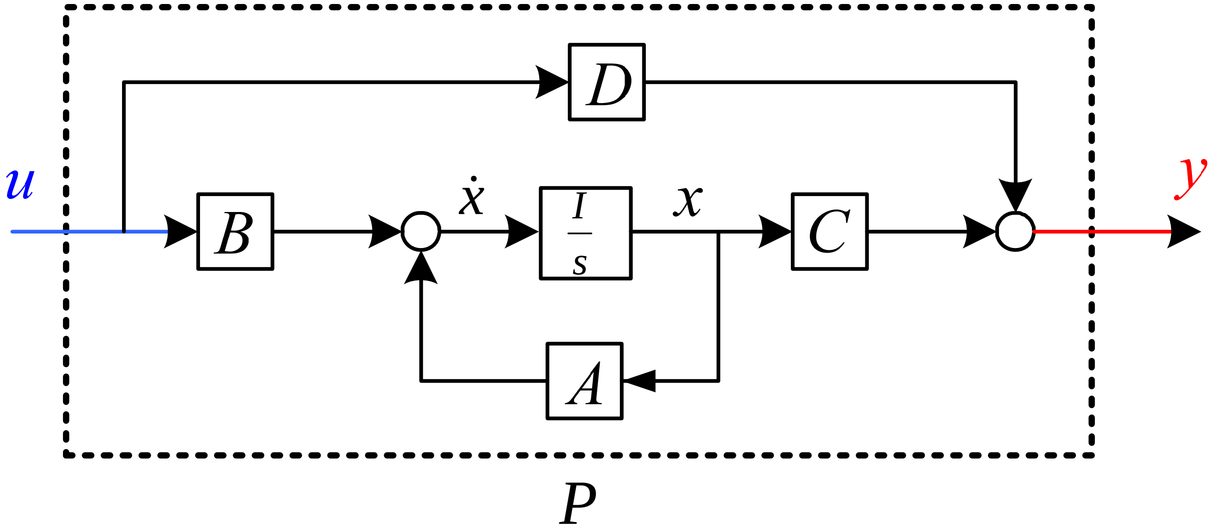
calculate $G(s) = P_1(s)P_2(s) \overset{s}{=} \left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array}\right]\left[\begin{array}{c|c} A_2 & B_2 \\ \hline C_2 & D_2 \end{array}\right]$ **as a general matrix manipulation.**

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \neq \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

system matrix

<State-space formula for inversion of a system>

Let $y = P(s)\cdot u$ with $P(s) \overset{s}{=} \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$



Question : How to find $P^{-1}(s)$

<I> Inversion

<Traditional approach>

by

$$(A_1 + A_2 \cdot A_3 \cdot A_4)^{-1} = A_1^{-1} - A_1^{-1} \cdot A_2 \cdot (A_4 \cdot A_1^{-1} \cdot A_2 + A_3^{-1})^{-1} \cdot A_4 \cdot A_1^{-1}$$

and

$$\frac{y}{u} = P(s) = D + C \cdot (sI - A)^{-1} \cdot B$$

one has

$$\Rightarrow P^{-1} = [D + C \cdot (sI - A)^{-1} \cdot B]^{-1}$$
$$= D^{-1} - D^{-1} \cdot C \cdot [B \cdot D^{-1} \cdot C + (sI - A)]^{-1} \cdot B \cdot D^{-1}$$
$$= D^{-1} - (D^{-1}C) \cdot [sI - (A - BD^{-1}C)]^{-1} \cdot (BD^{-1})$$
$$= D_{inv} + C_{inv} \cdot (sI - A_{inv})^{-1} \cdot B_{inv}$$

$$\Rightarrow P^{-1}(s) \overset{s}{=} \left[\begin{array}{c|c} A_{inv} & B_{inv} \\ \hline C_{inv} & D_{inv} \end{array}\right] = \left[\begin{array}{c|c} A - BD^{-1}C & BD^{-1} \\ \hline -D^{-1}C & D^{-1} \end{array}\right]$$