

# Robust and Optimal Control

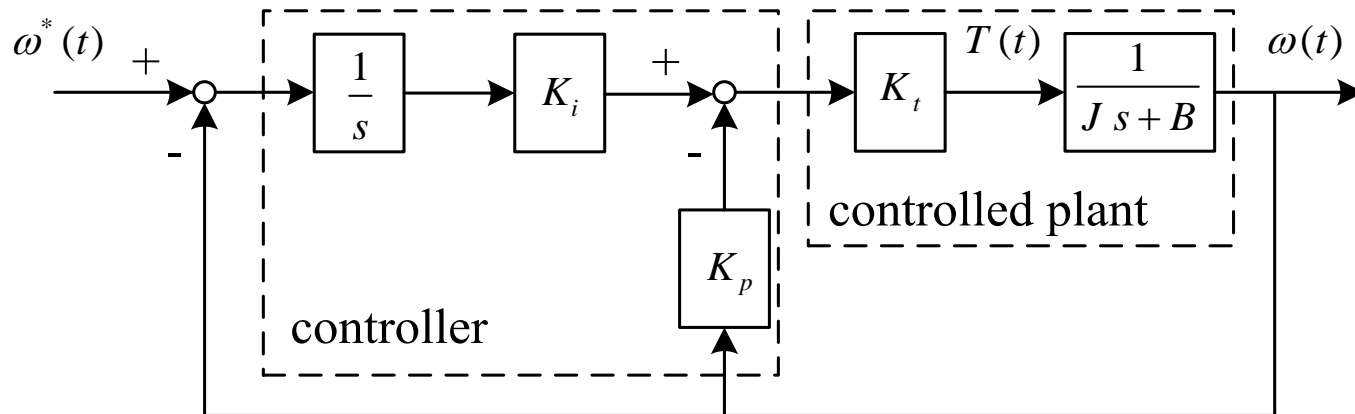
## A Two-port Framework Approach

# Design Example

***Classical PDF  
Controller Design***

## Classical PDF Controller Design

Speed control with classical PDF controller



**closed-loop transfer function:** 
$$T(s) = \frac{\omega(s)}{\omega^*(s)} = \frac{(K_i K_t / J)}{s^2 + [(B + K_p K_t) / J]s + (K_i K_t / J)}$$

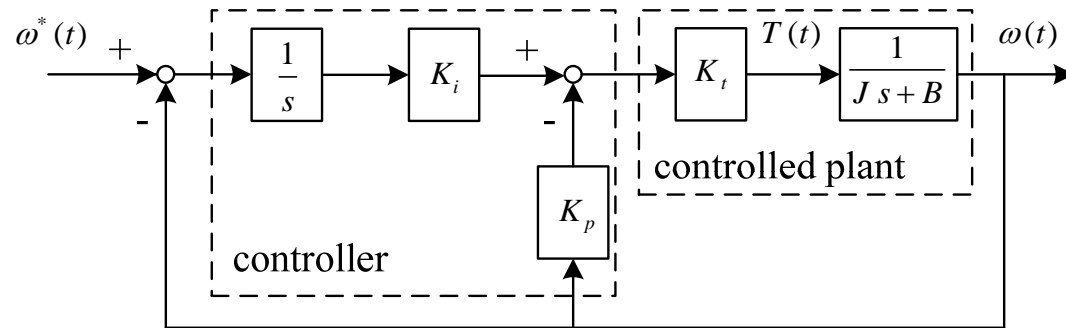
conventional controller design: coefficient comparison

damping ratio :  $\zeta$       bandwidth :  $BW = \omega_n (1 - 2\zeta^2 + \sqrt{2 - 4\zeta^2 + 4\zeta^4})^{0.5}$

↳ 
$$K_I = \frac{J \omega_n^2}{K_t} \quad \& \quad K_P = \frac{2J\zeta\omega_n - B}{K_t}$$

For the **nominal model**, the controller design could be finished easily.

# Classical PDF Controller Design



moment of inertia :  $J = 5.77 \times 10^{-5}$

viscous friction constant :  $B = 0.55 \times 10^{-3}$

torque constant :  $K_t = 0.21$

For  $\zeta = 0.9$  &  $B.W. = 100$  Hz , the PDF controller parameters are determined by

$$K_i = J \omega_n^2 / K_t = 194.74$$

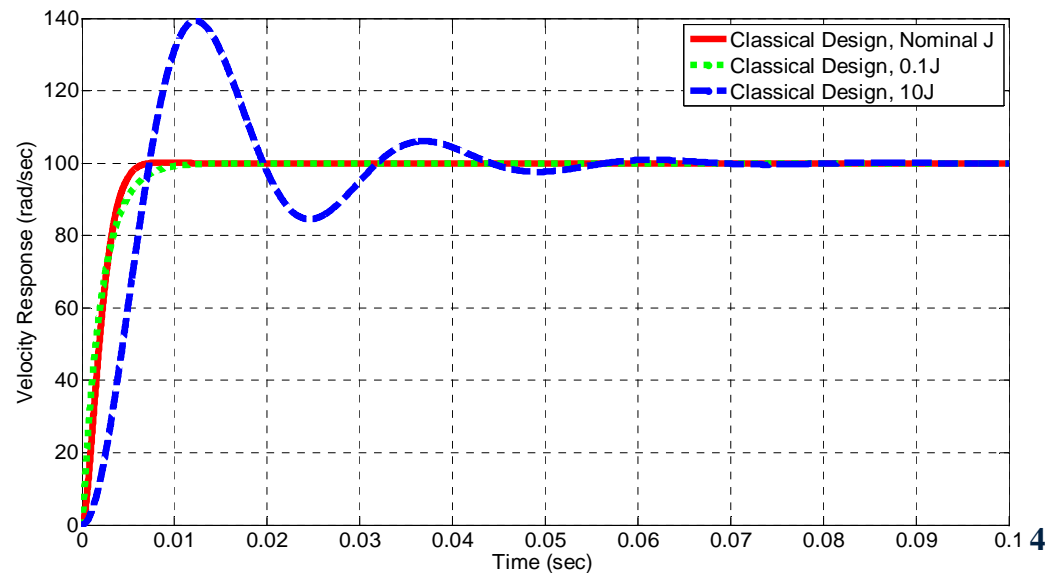
$$K_p = (2J\zeta\omega_n - B) / K_t = 0.41$$

The effect of model uncertainty:

$$J = 0.1J_o$$

$$J = J_o$$

$$J = 10J_o$$



# Classical PDF Controller Design

Characteristic equation:

$$Js^2 + (B + K_p K_t)s + K_t K_i = 0 \quad \longrightarrow \quad 1 + kL(s) = 0, \quad L(s) = \frac{(B + K_p K_t)s + K_t K_i}{s^2}, \quad k = \frac{1}{J}$$

Root locus plot according to variation of  $\frac{1}{J}$

