Chapter 6

Problem 1.

Derive a stable coprime factorization of the system with an inner denominator

$$T(s) \overset{s}{=} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 1 & 2 & 0 \\ \hline 0 & 1 & 0 \end{array}\right]$$

Step1:

$$T(s) = \frac{1}{s^2 - 3s + 2}$$

Choose $F = [a \quad b]$, and $A + BF$ must be Hurwitz matrix.

If $F = [-6 \quad -12] \Rightarrow A + BF = \begin{bmatrix} -5 & -12 \\ 1 & 2 \end{bmatrix}$

The eigenvalues of $A + BF$ are [-1 -2] < 0.

Hence, $A + BF$ is Hurwitz.

Step 2:

$$M(s) \overset{s}{=} \left[\begin{array}{c|c} A+BF & BW \\ \hline F & W \end{array}\right], \; N(s) \overset{s}{=} \left[\begin{array}{c|c} A+BF & BW \\ \hline C+DF & DW \end{array}\right]$$

As W is invertible, choose $W = 1$.

Hence,

$$M \overset{s}{=} \left[\begin{array}{cc|c} -5 & -12 & 1 \\ 1 & 2 & 0 \\ \hline -6 & -12 & 1 \end{array}\right] \overset{s}{=} \frac{s^2 - 3s + 2}{s^2 + 3s + 2},$$

$$N \overset{s}{=} \left[\begin{array}{cc|c} -5 & -12 & 1 \\ 1 & 2 & 0 \\ \hline 0 & 1 & 0 \end{array}\right] \overset{s}{=} \frac{1}{s^2 + 3s + 2}.$$