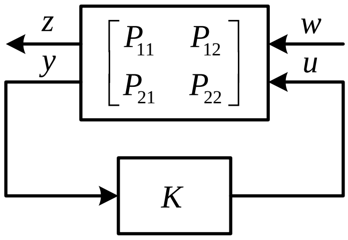
Chapter 5

Problem 1

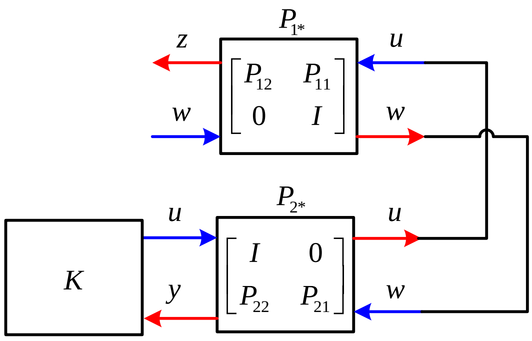
Let P be an SCC plant shown below, where $P(s)=\begin{bmatrix}\frac{1}{s+1} & \frac{1}{s(s+1)}\\ 1 & \frac{1}{s}\end{bmatrix}$, $K=-3$.



(a) Find the closed-loop transfer function matrix of $LFT_l(P,K)$.

(b) Transform the SCC plant P into a right CSD matrix G, and calculate the transfer function of $CSD_r(G,K)$.

(c) Let P be represented by a right CSD matrix P_{1*}, associated with a left CSD matrix P_{2*} as in the following figure. Find P_{1*} and P_{2*}, and determine $CSD_r\left(P_{1*},CSD_l\left(P_{2*},K\right)\right)$.



(a)

$$LFT(P,K)=P_{11}+P_{12}\left(I-KP_{22}\right)^{-1}KP_{21}$$

$$=\frac{1}{s+1}+\left(\frac{1}{s(s+1)}\right)\left(1+\frac{3}{s}\right)^{-1}\cdot -3\cdot 1$$

$$=\frac{s}{(s+1)(s+3)}$$

(b)

From the formula for transferring SCC-matrix to right CSD-matrix

$$G=\begin{bmatrix} P_{12}-P_{11}P_{21}{}^{-1}P_{22} & P_{11}P_{21}{}^{-1} \\ -P_{21}{}^{-1}P_{22} & P_{21}{}^{-1} \end{bmatrix}$$

$$=\begin{bmatrix} \left(\frac{1}{s(s+1)}\right)-\left(\frac{1}{s+1}\right)1^{-1}\left(\frac{1}{s}\right) & \left(\frac{1}{s+1}\right)1^{-1} \\ -1^{-1}\left(\frac{1}{s}\right) & 1^{-1} \end{bmatrix}$$

$$=\begin{bmatrix} 0 & \frac{1}{s+1} \\ -\frac{1}{s} & 1 \end{bmatrix}$$

thus, the right chain scattering description can be represented as

$$\begin{aligned} CSD_r(G,K) &= (G_{11}K+G_{12})(G_{21}K+G_{22})^{-1} \\ &= \left(0\cdot -3+\frac{1}{s+1}\right)\left(-\frac{1}{s}\cdot -3+1\right)^{-1} \\ &= \left(\frac{1}{(s+1)}\right)\left(\frac{s}{(s+3)}\right) \\ &= \frac{s}{(s+1)(s+3)} \end{aligned}$$

(c)

The P_{1*} and P_{2*} is as following:

$$P_{1^*}=\begin{bmatrix} P_{12} & P_{11} \\ 0 & I \end{bmatrix}=\begin{bmatrix} \frac{1}{s(s+1)} & \frac{1}{s+1} \\ 0 & I \end{bmatrix} \quad K=-3$$

$$P_{2^*}=\begin{bmatrix} I & 0 \\ P_{22} & P_{21} \end{bmatrix}=\begin{bmatrix} I & 0 \\ \frac{1}{s} & 1 \end{bmatrix}$$

From P_{2*} and K , transfer function of $CSD_l(P_{2*},K)$ can be calculated as

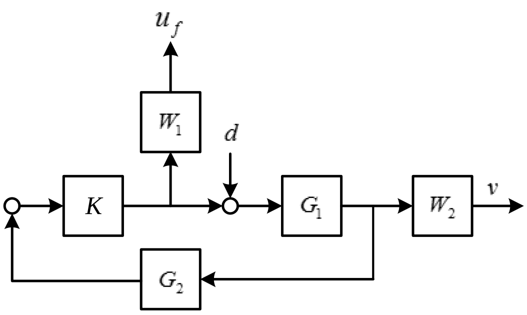
$$CSD_l(P_{2*},K)=-\left(I-K\times\frac{1}{s}\right)^{-1}(0-K\times 1)=-3\frac{s}{s+3}$$

And the transfer function of $CSD_r\left(P_{1*},CSD_l\left(P_{2*},K\right)\right)$ is obtained as

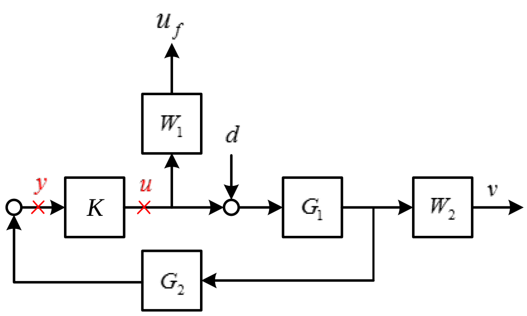
$$CSD_r\left(P_{1*},CSD_l\left(P_{2*},K\right)\right)=\left(\frac{1}{s(s+1)}\times -3\frac{s}{s+3}+\frac{1}{s+1}\right)\left(0\times -3\frac{s}{s+3}+1\right)^{-1}=\frac{s}{(s+1)(s+3)}$$

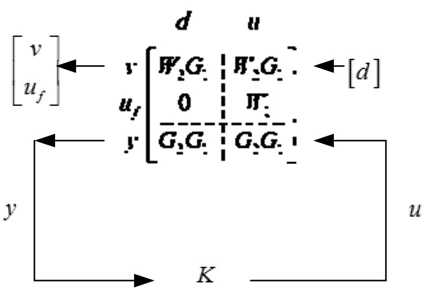
Problem2

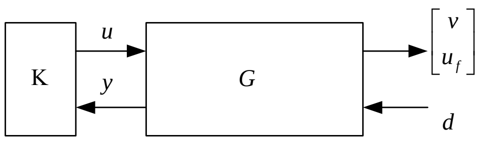
Determine the interconnected matrix P in the SCC plant for the following system, where $w = \begin{bmatrix} n \\ d \end{bmatrix}$, $z = \begin{bmatrix} v \\ u_f \end{bmatrix}$ and transform it into a CSD form.



Step 1: Cut the system as shown below.







Step 2: Calculate the matrix P and matrix K.

$$P = \begin{matrix} & d & u \\ v & W_2G_1 & W_2G_1 \\ u_f & 0 & W_1 \\ y & G_2G_1 & G_2G_1 \end{matrix}, \quad P = \left[\begin{array}{c|c} W_2G_1 & W_2G_1 \\ 0 & W_1 \\ \hline G_2G_1 & G_2G_1 \end{array}\right]$$

$$P_{11} = \begin{bmatrix} W_2G_1 \\ 0 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} W_2G_1 \\ W_1 \end{bmatrix}, \quad P_{21} = [G_2G_1], \quad P_{22} = [G_2G_1], \quad \mathrm{K} = K$$