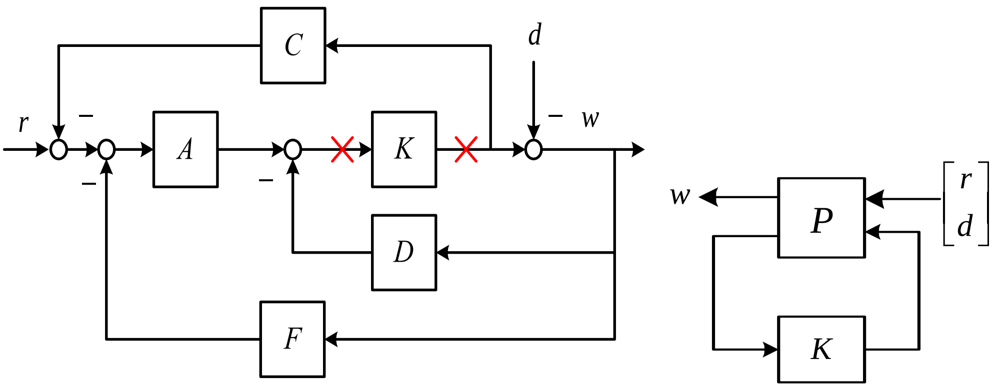
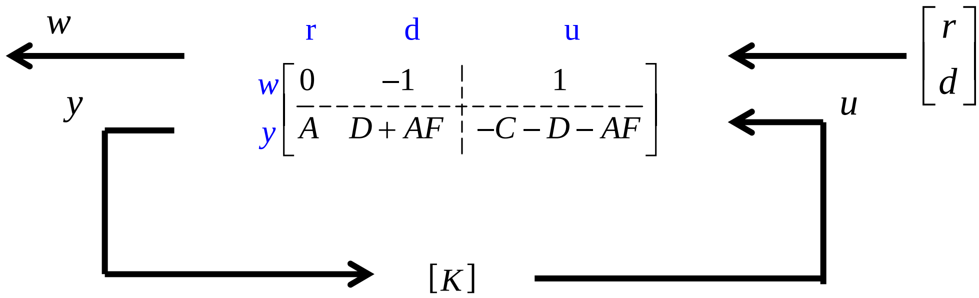
Chapter 4

Problem 1

Let $w = LFT_l(P,K)\begin{bmatrix} r \\ d \end{bmatrix}$ in the block-diagram below. Determine P and $LFT_l(P,K)$ with the given cutting point.





$$LFT_l(P,K) = P_{11} + P_{12}K\left(I - P_{22}K\right)^{-1}P_{21}$$

$$LFT_l(P,K) = \begin{bmatrix} 0 & -1 \end{bmatrix} + K\left(1 + CK + DK + AFK\right)^{-1}\begin{bmatrix} A & D + AF \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \end{bmatrix} + \frac{K\begin{bmatrix} A & D + AF \end{bmatrix}}{\left(1 + CK + DK + AFK\right)}$$

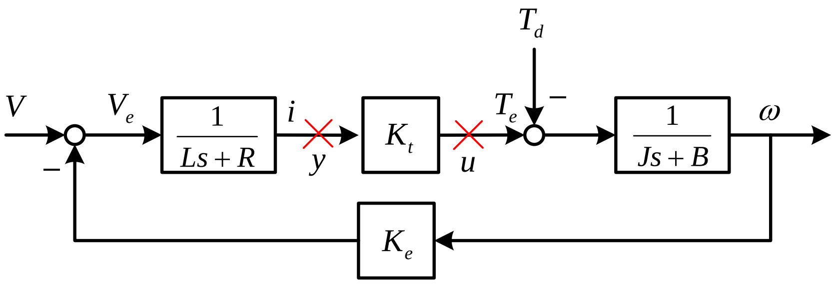
$$P = \left[\begin{array}{cc|c} 0 & -1 & 1 \\ \hline A & D + AF & -C - D - AF \end{array}\right]$$

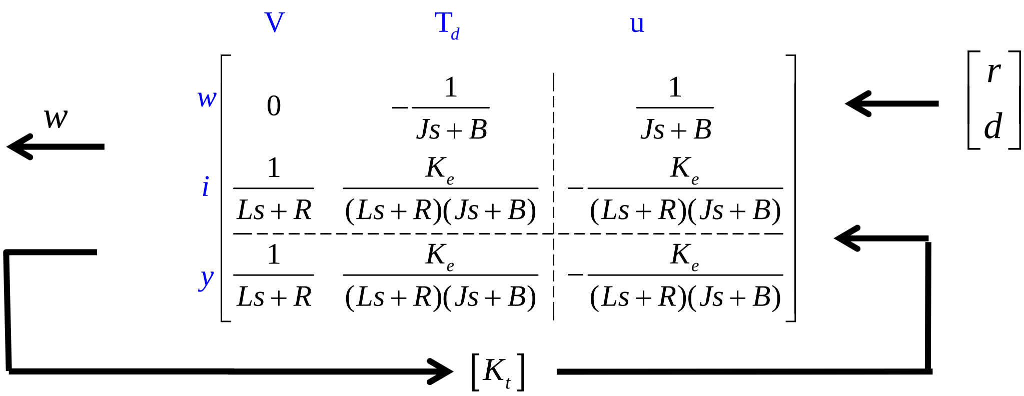
$$LFT_l(P,K) = \begin{bmatrix} 0 & -1 \end{bmatrix} + \frac{K}{(1+CK+DK+AFK)} \begin{bmatrix} A & D+AF \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{KA}{(1+CK+DK+AFK)} & \dfrac{-K(D+AF)}{(1+CK+DK+AFK)} \end{bmatrix}$$

Problem 2

Consider a brushed DC motor model as given below, and derive its corresponding LFT representation of $\begin{bmatrix} \omega \\ i \end{bmatrix} = P \begin{bmatrix} T_d \\ V \end{bmatrix}$, and $T_e = K_t i$.





$$\begin{matrix} & V & T_d & u \\ w & 0 & -\dfrac{1}{Js+B} & \dfrac{1}{Js+B} \\ i & \dfrac{1}{Ls+R} & \dfrac{K_e}{(Ls+R)(Js+B)} & -\dfrac{K_e}{(Ls+R)(Js+B)} \\ y & \dfrac{1}{Ls+R} & \dfrac{K_e}{(Ls+R)(Js+B)} & -\dfrac{K_e}{(Ls+R)(Js+B)} \end{matrix}$$

$$LFT_l(P,K) = P_{11} + P_{12}K\left(I - P_{22}K\right)^{-1}P_{21}$$

$$= \begin{bmatrix} 0 & -\dfrac{1}{Js+B} \\ \dfrac{1}{Ls+R} & \dfrac{K_e}{(Ls+R)(Js+B)} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{Js+B} \\ -\dfrac{K_e}{(Ls+R)(Js+B)} \end{bmatrix} [K_t]$$

$$\left(1 + \frac{K_e}{(Ls+R)(Js+B)}K_t\right)^{-1} \begin{bmatrix} \dfrac{1}{Ls+R} & \dfrac{K_e}{(Ls+R)(Js+B)} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\dfrac{1}{Js+B} \\ \dfrac{1}{Ls+R} & \dfrac{K_e}{(Ls+R)(Js+B)} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{Js+B} \\ -\dfrac{K_e}{(Ls+R)(Js+B)} \end{bmatrix} [K_t]$$

$$\begin{bmatrix} \dfrac{(Ls+R)(Js+B)}{((Ls+R)(Js+B)+K_eK_t)(Ls+R)} & \dfrac{(Ls+R)(Js+B)K_e}{((Ls+R)(Js+B)+K_eK_t)(Ls+R)(Js+B)} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\dfrac{1}{Js+B} \\ \dfrac{1}{Ls+R} & \dfrac{K_e}{(Ls+R)(Js+B)} \end{bmatrix} +$$

$$\begin{bmatrix} \dfrac{K_t}{JLs^2+(JR+BL)s+BR+K_eK_t} & \dfrac{K_eK_t}{((Ls+R)(Js+B)+K_eK_t)(Js+B)} \\ -\dfrac{K_tK_e}{((Ls+R)(Js+B)+K_eK_t)(Ls+R)} & -\dfrac{K_e{}^2K_t}{((Ls+R)(Js+B)+K_eK_t)(Ls+R)(Js+B)} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{K_t}{JLs^2+(JR+BL)s+BR+K_eK_t} & \dfrac{-(R+Ls)}{JLs^2+(JR+BL)s+BR+K_eK_t} \\ \dfrac{(B+Js)}{JLs^2+(JR+BL)s+BR+K_eK_t} & \dfrac{K_e}{JLs^2+(JR+BL)s+BR+K_eK_t} \end{bmatrix}$$