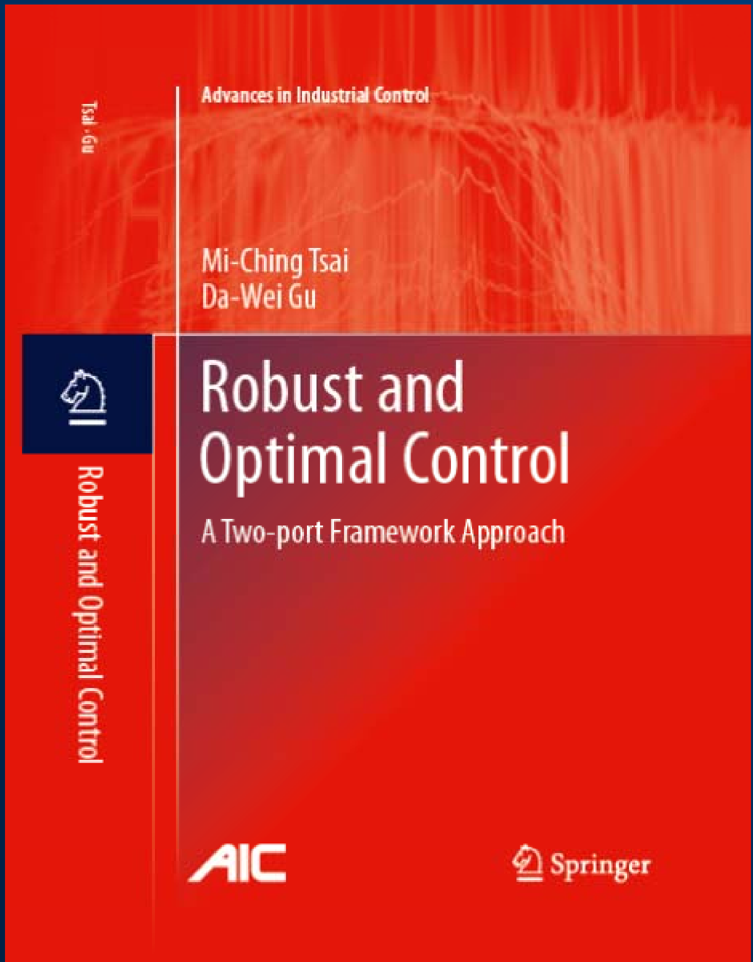


Robust and Optimal Control

A Two-port Framework Approach

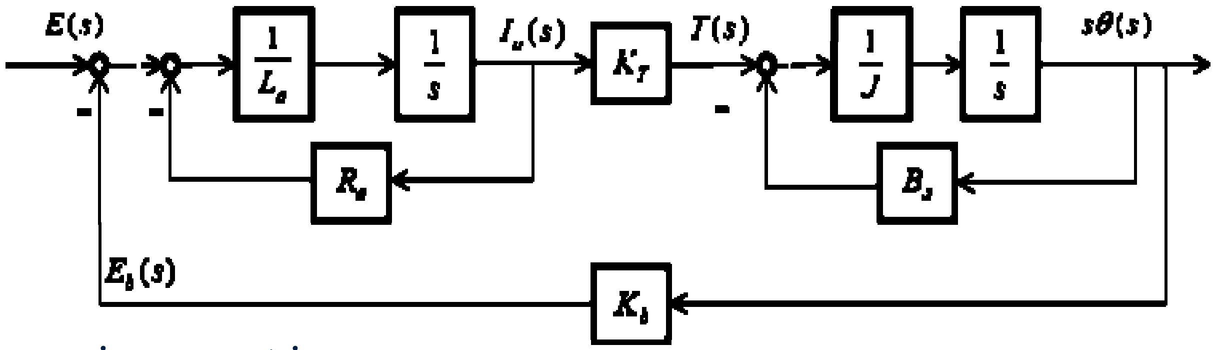
CSD Approach to Optimal H_2 Controllers

Purpose of controller：

1. Control the rotational degree of the servo motor as the input.
2. Minimize the supply voltage to achieve the target.

$$J = \min \int_0^\infty \left(y^T Q y + u^T R u \right) dt$$

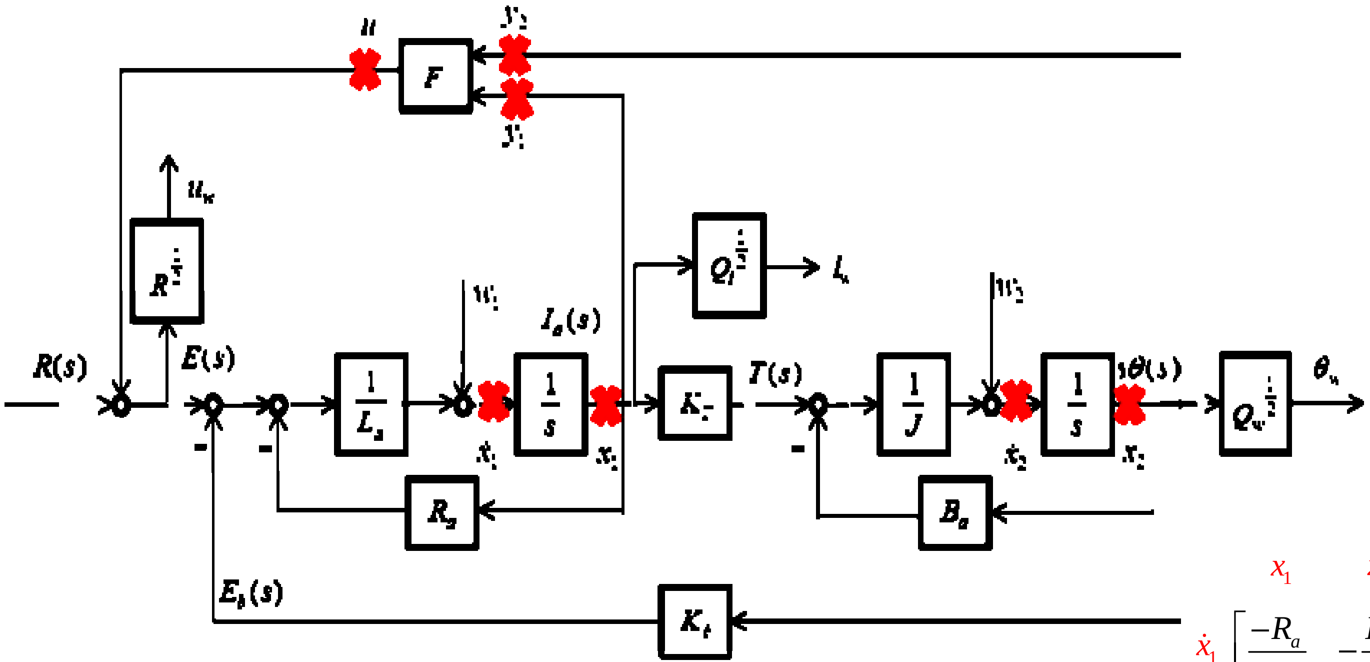
Model of servo motor：



Dynamic equation：

$$\begin{cases} V = K_b \dfrac{d\theta}{dt} + L\dfrac{di}{dt} + iR_a \\ J\dfrac{d^2\theta}{dt^2} = -B_a \dfrac{d\theta}{dt} + K_t i \end{cases}$$

The block diagram of servo motor for ***full state feedback control***：



Initial condition：

$$\begin{cases} w_1 = 0 \\ w_2 = 100 rad/s \end{cases}$$

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \stackrel{s}{=} \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \\ C_2 & 0 & 0 \end{array}\right] =$$

$$\begin{array}{c} \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{\theta}_w \\ i_w \\ u_w \\ y_1 \\ y_2 \end{array}
\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & w_1 & w_2 & u \end{array} \\
\left[\begin{array}{cc|cc:c} \frac{-R_a}{L_a} & -\frac{K_b}{L_a} & 1 & 0 & \frac{1}{L_a} \\ \frac{K_T}{J} & -\frac{B_a}{J} & 0 & 1 & 0 \\ \hline 0 & Q_W^{\frac{1}{2}} & 0 & 0 & 0 \\ Q_i^{\frac{1}{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R^{\frac{1}{2}} \\ \hdashline 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{array}\right] \end{array}$$

From the following list, we can seen that the P(s) is similar as State Feedback(SF) case, so that the special case of State Feedback will be used to solve our problem.

$$P(s)=\begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \overset{s}{=} \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \\ I & 0 & 0 \end{array}\right]$$

Formulation of System	Corresponding Plant	Formulation of System	Corresponding Plant
Disturbance Feedforward (DF)	$P_{DF}(s)\overset{s}{=}\left[\begin{array}{c\|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & I & 0 \end{array}\right]$ with $(A-B_1C_2)$ is Hurwitz	**Output Estimation (OE)**	$P_{OE}(s)\overset{s}{=}\left[\begin{array}{c\|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & I \\ C_2 & D_{21} & 0 \end{array}\right]$ with $(A-B_2C_1)$ is Hurwitz
Full Information (FI)	$P_{FI}(s)\overset{s}{=}\left[\begin{array}{c\|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ \begin{bmatrix} I \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ I \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{array}\right]$	**Full Control (FC)**	$P_{FC}(s)\overset{s}{=}\left[\begin{array}{c\|cc} A & B_1 & \begin{bmatrix} I & 0 \end{bmatrix} \\ \hline C_1 & D_{11} & \begin{bmatrix} 0 & I \end{bmatrix} \\ C_2 & D_{21} & \begin{bmatrix} 0 & 0 \end{bmatrix} \end{array}\right]$
State Feedback (SF)	$P_{SF}(s)\overset{s}{=}\left[\begin{array}{c\|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ I & 0 & 0 \end{array}\right]$	**Output Injection (OI)**	$P_{OI}(s)\overset{s}{=}\left[\begin{array}{c\|cc} A & B_1 & I \\ \hline C_1 & D_{11} & 0 \\ C_2 & D_{21} & 0 \end{array}\right]$