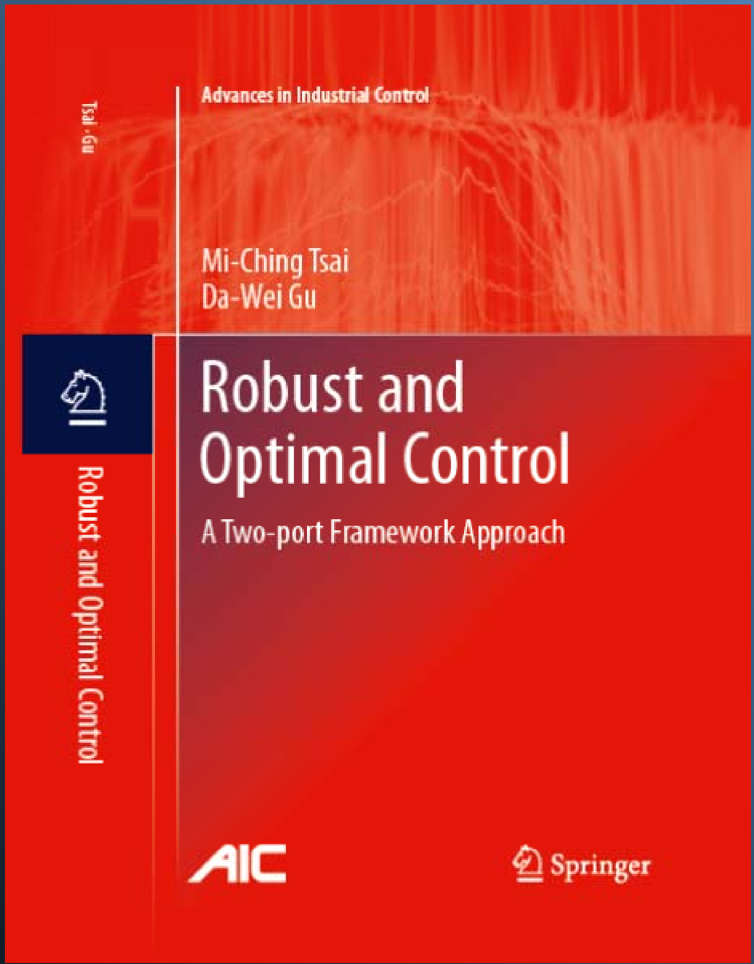

Robust and Optimal Control

A Two-port Framework Approach

State-space Formulae for Spectral Factorization

Outline

- Spectral factorizations
- Case I to III
- Examples

Spectral Factorization

Definition 1:

Consider a square matrix $\Omega(s)$ *having the properties:*

$$\Omega \in RL_\infty,\ \Omega^{-1} \in RL_\infty,\ \Omega^{\sim} = \Omega, \text{ and } \Omega(\infty) > 0$$

Then, $\Omega = \Phi^{\sim}\Phi$

is called the spectral factorization of $\Omega(s)$ *, and* $\Phi \in GH_\infty$ *is called spectral factor.*

Both $\Phi(s)$ *and its inverse are stable.*

Note :

1. **Because** $\Omega = \Phi^{\sim}\Phi$, $\Omega(s)$ **has poles and zeros in symmetry about the imaginary axis.**
2. **The spectral factor** $\Phi(s)$ **is outer.**

Spectral Factorization

Definition 2:

Let $\Sigma = \Sigma^T$ *and* $\breve{\Sigma} = \breve{\Sigma}^T$ *be constant matrices with compatible dimensions. Then* $P(s) \in RL_\infty$ *satisfying* $P^{\sim}(s)\Sigma P(s) = \breve{\Sigma}$ *is defined as* ***weighted all-pass***

Lemma *Let* $P(s) \overset{s}{=} \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \in RL_\infty$. *Given* $\Sigma = \Sigma^T$ *and* $\breve{\Sigma} = \breve{\Sigma}^T$

If $D^T \Sigma D = \breve{\Sigma}$

and there exists a matrix $X = X^T \geq 0$ *such that*

$$XB + C^T \Sigma D = 0$$

$$A^T X + XA + C^T \Sigma C = 0$$

then

$$P^{\sim}(s)\Sigma P(s) = \breve{\Sigma}$$ **weighted all-pass**