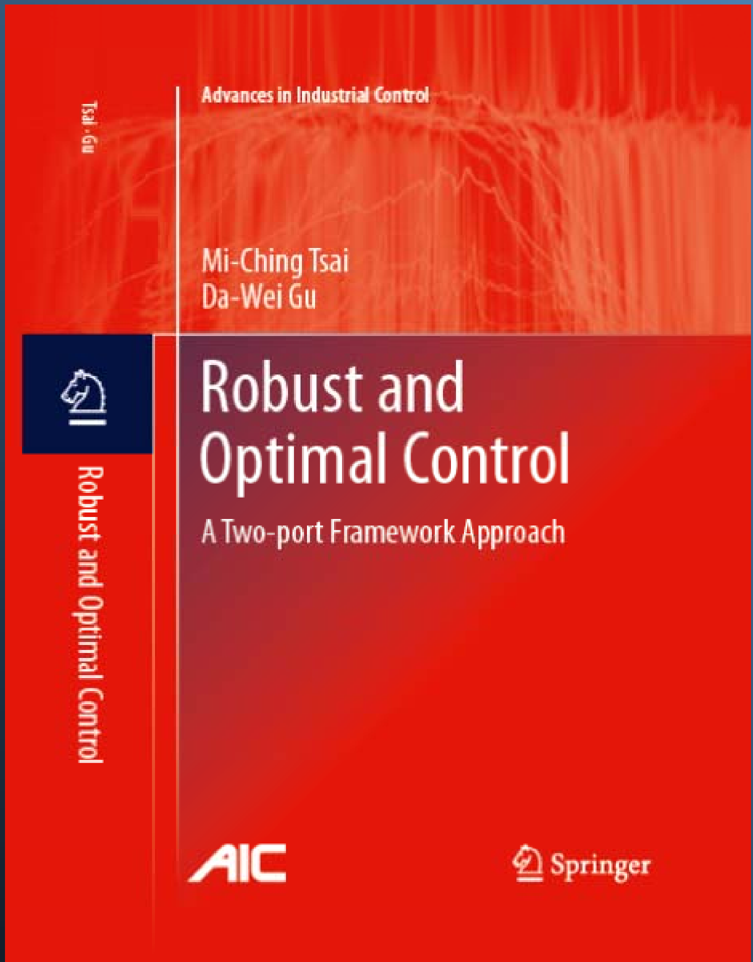
Robust and Optimal Control

A Two-port Framework Approach



Examples of Useful Mathematic Tools and Examples of norm

Content

Examples of norm

- Vector norm
- Matrix norm

Example of normal matrix

- Eigenvalue
- Singular values decomposition

Example of norm

Example of vector norm

$$a = \begin{bmatrix} 1 \\ 2 \\ 15 \end{bmatrix} \quad b = \begin{bmatrix} 8 \\ 9 \\ 10 \end{bmatrix} \quad c = \begin{bmatrix} 4 \\ -6 \\ -12 \end{bmatrix}$$

Consider three vectors a, b, c, please calculated vector norm by hand then verified the answer by Matlab.

(Include 1-norm, 2-norm, ∞-norm)

Definition of vector p-norm: (for 1≦p ≦ ∞) $\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p}$

1-norm of vector

Definition : *The 1-norm of any vextor **x** is defined as* $\|x\|_1 = \sum_{i=1}^{n} |x_i|$

$$\|a\|_1 = \sum_{i=1}^{n} |a_i| = |1| + |2| + |15| = 18 \qquad \|b\|_1 = \sum_{i=1}^{n} |b_i| = |8| + |9| + |10| = 16$$

$$\|c\|_1 = \sum_{i=1}^{n} |c_i| = |4| + |-6| + |-12| = 22$$

2-norm & ∞-norm of vector

2-norm of vector

Definition : *The 1-norm of any vextor* $\mathbf{x}$ *is defined as*

$$\|x\|_2 = \sqrt{x^* x} = \sqrt{\sum_{i=1}^{n} |x_i|^2}$$

$$\|a\|_2 = \sqrt{a^* a} = \sqrt{\sum_{i=1}^{n} |a_i|^2} = \sqrt{|1|^2 + |2|^2 + |15|^2} = \sqrt{230}$$

$$\|b\|_2 = \sqrt{b^* b} = \sqrt{\sum_{i=1}^{n} |b_i|^2} = \sqrt{|8|^2 + |9|^2 + |10|^2} = \sqrt{245}$$

$$\|c\|_2 = \sqrt{c^* c} = \sqrt{\sum_{i=1}^{n} |c_i|^2} = \sqrt{|4|^2 + |6|^2 + |-12|^2} = \sqrt{196} = 14$$

∞ -norm of vector

Definition : *The ∞-norm of any vextor* $\mathbf{x}$ *is defined as*

$$\|x\|_\infty = \max_{1 \le i \le n} |x_i|$$

$$\|a\|_\infty = |15| = 15 \qquad \|b\|_\infty = |10| = 10 \qquad \|c\|_\infty = |-12| = 12$$