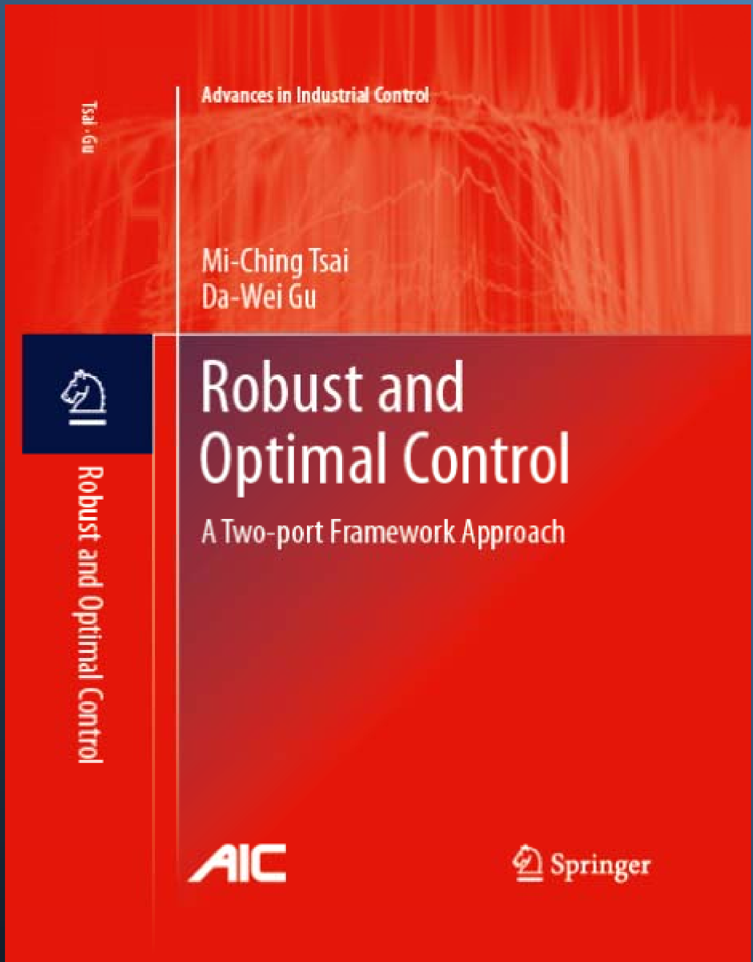

Robust and Optimal Control

A Two-port Framework Approach

PDF Control of DC Motor

Content

- Introduction
- Methodology
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- Discussion
- Conclusion

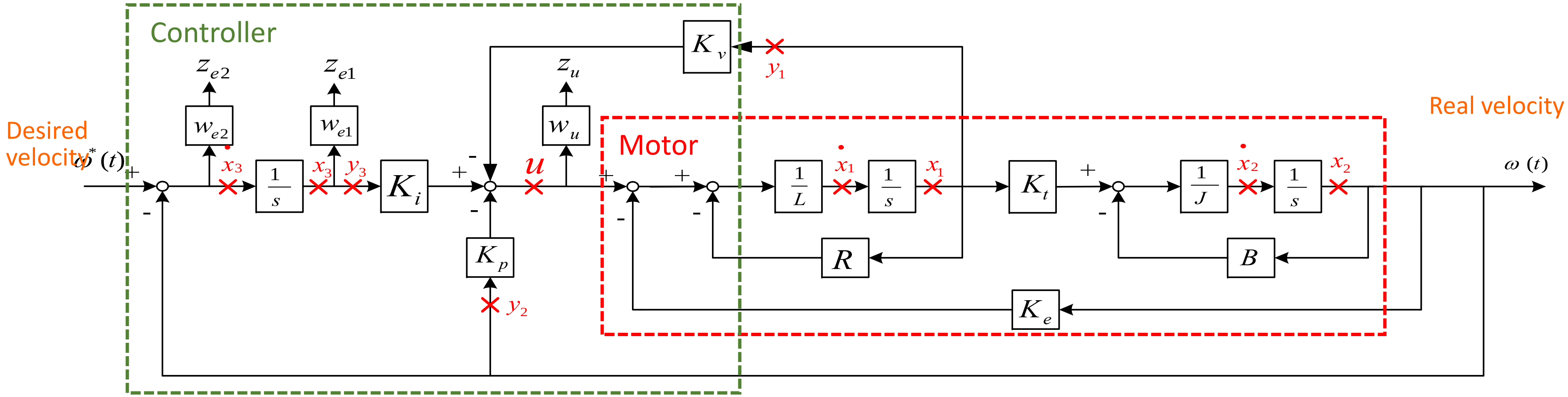
Introduction

1) Purpose: Design a controller for velocity control of servo motor
2) Target: Minimum the tracking error
3) Controller: PDF Controller

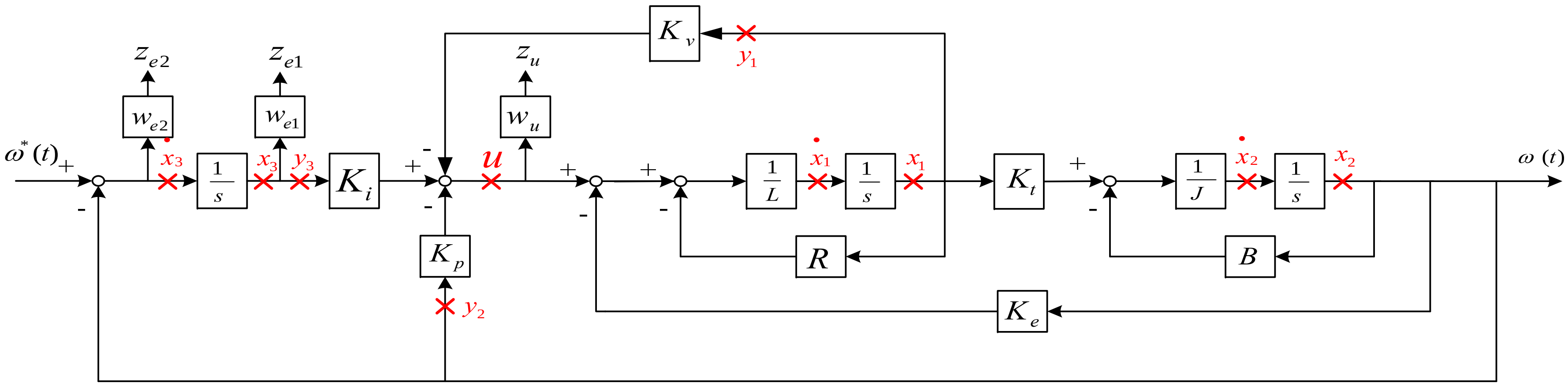
Dynamic equation of the motor:

$$\begin{cases} V = iR_c + L\dfrac{di}{dt} + k_e\omega \\ J_m\dfrac{d\omega}{dt} = k_t i - B_m\omega \end{cases}$$

Block diagram of velocity control:



Step1.Find LFT matrix P(s)



Simulation parameters:

Resistance, R_c	7.155
Inductance, L	0.0038
Inertia of motor, J_m	$5.77\cdot10^{-5}$
Damping ratio, B_m	0.00055
Back EMF constant, k_e	0.21
Torque constant, k_t	0.21

Linear fractional transformation:

$$P=\begin{bmatrix}\frac{1}{\gamma}P_{11} & \frac{1}{\gamma}P_{12}\\ P_{21} & P_{22}\end{bmatrix}\overset{s}{=}\begin{array}{c}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ z_{e1}\\ z_{e2}\\ z_u\\ y_1\\ y_2\\ y_3\end{array}\left[\begin{array}{ccc|c|c} x_1 & x_2 & x_3 & \omega^* & u\\ \hline -\frac{R}{L} & -\frac{K_e}{L} & 0 & 0 & \frac{1}{L}\\ \frac{K_t}{J} & -\frac{B}{J} & 0 & 0 & 0\\ 0 & -1 & 0 & 1 & 0\\ \hline 0 & 0 & \frac{1}{\gamma}W_{e1} & 0 & 0\\ 0 & \frac{1}{\gamma}W_{e2} & 0 & \frac{1}{\gamma}W_{e2} & 0\\ 0 & 0 & 0 & 0 & \frac{1}{\gamma}W_u\\ \hline 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\end{array}\right]$$

Step2: Transformation for LFT-matrix to CSD-matrix

γ Condition:

$\left\| LFT_l(\mathrm{P}, K_\infty) \right\|_\infty < \gamma$

$\Rightarrow \gamma = 2$

$$P = \begin{bmatrix} \frac{1}{\gamma} P_{11} & \frac{1}{\gamma} P_{12} \\ P_{21} & P_{22} \end{bmatrix} \overset{s}{=} \begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \hline z_{e1} \\ z_{e2} \\ z_u \\ \hline y_1 \\ y_2 \\ y_3 \end{array} \left[\begin{array}{ccc|c:c} -\frac{R}{L} & -\frac{K_e}{L} & 0 & 0 & \frac{1}{L} \\ \frac{K_t}{J} & -\frac{B}{J} & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ \hline 0 & 0 & \frac{1}{\gamma} W_{e1} & 0 & 0 \\ 0 & \frac{1}{\gamma} W_{e2} & 0 & \frac{1}{\gamma} W_{e2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\gamma} W_u \\ \hdashline 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

Check condition:

1. Is (A, B_2) stabilizable ?

- rank[λ_1I-A, B_2]=2
- rank[λ_2I-A, B_2]=2

$\Rightarrow$ **OK!**

2. Is (C_2,A) detectable?

- rank[C_2, λ_1I-A]'=2
- rank[C_2, λ_2I-A]'=2

$\Rightarrow$ **OK!**

$$rank\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = dim(A) + rank(D_{21}) = 2$$

$$rank\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = dim(A) + rank(D_{12}) = 3$$

$\Rightarrow$ **OK!**